Optimal One-Shot Stream Scheduling for MIMO Links in a Single Collision Domain

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***Abstract*— This paper considers the problem of scheduling the maximum number of streams in a single time slot under the MIMO degrees of freedom (DOF) model, when all links are in a single collision domain. In the DOF model, nodes can use degrees of freedom provided by their antenna arrays to multiplex multiple streams on a single link and/or cancel interference between concurrently transmitting links. Given a set of links with data to transmit that are free of primary interference, we provide optimal constructions for both the case where only spatial reuse (from interference cancellation) is allowed and the case where both spatial reuse and spatial multiplexing can be done simultaneously. Our analysis allows deriving clean throughput performance trends when the number of available DOFs is arbitrarily increased. These trends show that combining spatial multiplexing with spatial reuse can arbitrarily increase throughput compared to spatial reuse only, and that close to two-fold throughput increases can be achieved compared to spatial multiplexing only. Finally, we show how the approach can be extended to deal with primary interference and optimally solve the one-shot stream scheduling problem for an arbitrary set of MIMO links in a single collision domain.**

I. Introduction

MIMO antenna technology promises to revolutionize wireless networks by substantially increasing link throughput, boosting signal strength, and increasing transmission range. The potential of MIMO is beginning to be realized in wireless LANs with the availability of 802.11n components. However, the use and optimization of MIMO links in a broader network setting is only now beginning to be considered. In this paper, we consider how MIMO antennas with capability to cancel interference from competing links can be used to optimize network-wide performance. We focus on optimizing overall network throughput through the combination of spatial reuse and spatial multiplexing, enabled by the interference cancellation (IC) capability provided by MIMO.

MIMO can be used to achieve several capabilities, primarily individually but potentially in combination, as well. The individual capabilities provided by MIMO are: 1) spatial diversity, wherein the different paths between the multiple transmit and receive antenna elements are used to provide robust signal strength in the presence of fading in some of the paths, 2) spatial multiplexing, wherein the multiple paths are used to transmit a linear combination of multiple independent data streams that can then be individually decoded from the multiple signals provided by the receive antenna elements, and 3) spatial reuse, wherein multiple MIMO links that would otherwise destructively interfere with each other can transmit concurrently by using some antenna elements to cancel interference from the competing links. Note that spatial diversity can have an indirect benefit on throughput by maintaining the higher data rates enabled by improved signal to noise ratios. However, in this paper, we focus on the throughput benefits that can be achieved from increased spatial reuse and spatial multiplexing.

The specific problem we consider is: given a set of MIMO links with data to transmit, how can a set of transmitting links be chosen and the antenna elements of those links be allocated between spatial multiplexing and interference cancellation (to permit spatial reuse) so as to maximize achievable throughput? We consider a setting wherein all links are in the same collision domain, i.e. if interference cancellation is not done, no two links will be able to transmit concurrently, and channel state information (CSI) for all links is available at both transmitters and receivers. Furthermore, we assume that each link has virtually infinite data to transmit, i.e., an arbitrary number of distinct, non-empty data streams can potentially be allocated on each link. This assumption is coherent with this paper's perspective of investigating the maximum possible network throughput. Within this setting, we solve the considered problem exactly. To be specific, we analytically derive upper bounds on the throughput increases that can be provided by spatial reuse and spatial multiplexing, both individually and in combination, and we provide specific designs that achieve these upper bounds exactly.

II. Related Work and Contribution

Prior work exists on the characterization of the capacity of wireless MIMO networks. Jafar [9] obtained an expression for the maximum throughput for a two user non-degenerate MIMO Gaussian interference channel with M_1, M_2 (respectively) antennas at transmitters $1, 2$ and N_1, N_2 antennas at the corresponding receivers. Here, perfect CSI is assumed at both receivers. The transmitters possess channel knowledge only of the communication links with which they are associated.

Another interesting work in this area is [2] where the authors characterize the benefits of cross-layer optimizations in interference limited wireless mesh networks with MIMO links. They formulate a framework where data routing at the protocol layer, link scheduling at the MAC layer and stream control at the physical layer can be jointly optimized for throughput maximization in the presence of interference, and then develop an efficient algorithm to solve the resulting throughput optimization problem subject to fairness constraints. An information theoretic approach is adopted in [5]-[8] where the capacity of a multi user Gaussian interference channel is bounded.

Work on MIMO broadcast includes [3], [16], where multiuser interference is cancelled at the transmitter by 'Dirty Paper Coding', which is of theoretical importance but is considered impractical due to high complexity. Prior work on interference cancellation (IC) of multiuser MIMO systems has mainly focused on the uplink [10], [12]. However, because of the need for inexpensive mobile units with low complexity of realization, closed loop MIMO systems have been studied where CSI is known at the transmitter of the base station. In related work, multi-user precoder designs [19], [4] can serve multiple mobile units over the same frequency in such a way that co-channel interference is mitigated.

The primary issue of IC is balancing the need for high received signal power for each user against the interference produced by the signal at other receivers. Several different IC algorithms exist. While the basic idea behind these is the same, namely the use of CSI to predict and then counteract the interference produced at each node in the network, they achieve different performance objectives. Typical performance criteria include zero-interference transmission, minimum transmit power subject to a minimum signal-to-interference plus noise ratio at each receiver, or maximum throughput subject to a given transmit power constraint. Two commonly implemented IC techniques are the minimum mean squared error (MMSE) and the zero forcing (ZF) beamformers. MMSE does not null those interferers which are below the noise floor, it merely ignores them. ZF instead completely nulls all interferers irrespective of their strengths.

For the greater part, existing literature assumes CSI to be available only at the receivers. Transmitters possess no CSI. In other cases, CSI is assumed at the receivers, with transmitters possessing CSI only of the specific communication link(s) they are associated with. This is a suitable model for cellular networks where fast channel dynamics and node mobility make it impractical for channel information to be fed back at the desired rate. Our model however, deviates from this trend by assuming perfect CSI of all communication links as well as of all interfering links at every receiver and transmitter. This is a reasonable assumption, e.g., for the backbone of a wireless mesh network, where nodes are fixed and channel conditions do not change rapidly. Periodic measurement and sharing of channel states by receiver nodes and feedback to every transmitter node is thus a feasible system design.

The work that is most closely related to ours is [6], where the authors study the throughput of a *multi-hop* wireless MIMO network. Similarly to our work, they assume that perfect CSI is available at both receivers and transmitters. The authors cast the problem of optimal throughput determination as an IP formulation, whose solution is upper bounded by the corresponding LP formulation. This upper bound is used to numerically characterize throughput performance under different MIMO usages (spatial reuse, spatial multiplexing, and their combination). Differently from the results reported in [6], our approach is fully analytical, and provides a simple closed-form expression for the optimal throughput. On the other hand, our approach considers a restricted, single-hop collision domain, while the results reported in [6] are more general and consider multi-hop flows. Quite interesting, our analytical results qualitatively confirm the main findings of the numerical evaluation reported in [6] in terms of the relative throughput gains when spatial reuse is combined with spatial multiplexing (see Section VI).

III. Background and System Model

An example of MIMO interference cancellation is done by the zero forcing (ZF) beamformer, which is a linear technique. The ZF beamformer [1] is given by the $M_R \times M_T$ matrix $C = (H^*H)^{-1}H^*$, where M_T is the number of antenna elements at transmitter, M_R is the number of antenna elements at receiver, H is the $M_R \times M_T$ channel matrix, and H^* is its conjugate transpose. Matrix C is used to either pre-process the transmit signal at the transmitter end, or to post-process the receive signal at the receiver end. In the former case, we say that the transmitter nulls interference at receivers, while in the latter case, we say that the receiver suppresses interference from transmitters.

There are three key benefits to using transmit and receive arrays in a communication link:

1) The ability to mitigate interference.
2) The ability to spatially multiplex several data streams onto the MIMO channel.
3) The ability to mitigate small scale fading (a.k.a. spatial diversity).

It is important to note that these benefits cannot be fully realized simultaneously through linear processing. An antenna array (either transmitter or receiver array) with linear processing that mitigates interference has a diminished capacity in the number of spatially multiplexed streams that it can decode and in its ability to combat fading, and vice-versa. For a transmit or receive array, this tradeoff is summarized by the conservation theorem, which states (see pg.548, [1])

$$\text{diversity order} = M - N_S - N_I + 1 \geq 1$$

Here, M is the number of antenna elements in the array (either transmitter or receiver array), N_S is the number of spatially multiplexed streams supported by the MIMO link, and N_I is the number of interferers (interfering transmit antenna elements) that must be suppressed by the array if it is a receiver or the number of interfering receive antenna elements at which its signal must be nulled if it is a transmitter. Note that the number of degrees of freedom (DOF) that must be used to handle interference is dependent on the number of streams being transmitted on the interfering (or interfered-with) link, rather than the number of streams being transmitted on the array's own MIMO link.

Since the diversity order should be at least one, we have

$$M \geq N_S + N_I$$

The size of an antenna array therefore must be at least equal to the sum of the number of data streams that it supports and the number of interfering streams that it mitigates. This corresponds to the so-called degrees of freedom (DOF) model [6], [13] , wherein antenna elements provide DOFs that can be divided arbitrarily between stream multiplexing and interference cancellation.

There is thus an evident symmetry between transmitters and receivers in terms of the usage of available degrees of freedom in supporting spatially multiplexed streams and in mitigating interference. This is seen by considering two interfering MIMO links each carrying s_1 and s_2 data streams respectively. Suppose that the transmitter of link 1, T_1, nulls itself at the receiver of link 2, R_2. In order to avoid putting any energy from T_1 at the interfering s_2 elements of R_2, T_1 requires s_2 degrees of freedom to project its transmit vector into the space that is orthogonal to the space spanned by the MRC weights of each of the s_2 selected antennas at R_2. The constraint on the size of T_1 is therefore $M_{T_1} \geq s_1 + s_2$. On the other hand, suppose that R_2 suppresses the s_1 data streams from T_1. This would impose s_1 constraints on the receive vector. The constraint on the size of R_2 is therefore $M_{R_2} \geq s_1 + s_2$. Hence, irrespective of whether interference cancellation is done by a transmitter or a receiver, the same constraint on the array size is felt.

Assumptions used in this paper are as follows:

1) Perfect CSI of communication and interfering links is available at all transmitters and receivers.
2) All links are in the same collision domain. By the same collision domain, we mean that any two links that are being used simultaneously will each cause the other's transmission to fail unless interference between them is canceled.
3) Transmitters and receivers are both capable of interference cancellation.
4) Interference cancellation is coordinated such that, for any link l_1 interfering with another link l_2, either the transmitter of l_1 nulls its signal at receiver elements of l_2 or the receiver of l_2 suppresses the signal from the transmitter of l_1, but not both.
5) Primary interference has been eliminated, i.e. the set of links with data to transmit is free of primary interference. (This assumption is removed in Section V-E.)
6) All links have identical transmit and receive arrays, in terms of their sizes and signal processing capabilities.

Several of these assumptions deserve discussion. As mentioned in Related Work, Assumption 1 is reasonable to achieve with periodic channel measurement in a slowly-changing environment, e.g. a network without mobility and a not too dynamic external environment. A good example of such an environment is a wireless mesh network backbone, where all nodes (mesh routers) are in fixed locations. Note that this assumption is unstated but implicit in all prior works that have used the DOF model, e.g. [6], [13]. Perfect interference cancellation, as assumed in the DOF model, is possible *only* with CSI for the interfering link.

While we deal herein with the single collision domain case (Assumption 2), our results could be used as a component of an analysis of a larger multihop network, where our component solution characterizes the optimal throughput achievable within a finite region constituting a single collision domain of the larger network.

Assumption 4 can be realized in several ways. One involves communicating control information between coordinating links and would add additional overhead resulting in degraded throughput relative to the results derived herein. However, we envision a spatial-reuse TDMA (STDMA) approach [11], which is a realistic possibility for small to medium scale mesh networks (c.f. 802.16 [7]), wherein all communications and interference cancellations are precomputed and allocated to specific communication slots. With periodic distribution of such a schedule to all nodes in the network, achievable throughput converges toward the results derived herein as the scheduling period increases. It is important to note that we do not assume node cooperation, as done for example in cooperative MIMO. In cooperative MIMO, nodes cooperate in their *data* transmissions. Here, we assume only that channel state information is disseminated in some way, but data transmissions are strictly pair-wise.

Finally, Assumption 6 is suitable for homogeneous wireless networks. Again, a good example would be a wireless mesh backbone, where all backbone nodes use the same hardware.

The following quantities are defined in developing the system model.

k	$=$	number of antenna elements at each transmit and receive array
l	$=$	number of active links
$\mathbf{s}$	$=$	$l \times 1$ throughput vector containing the number of data streams carried by each link
$w(A)$	$=$	weight of matrix or vector A (sum of all entries)
$\mathbf{s_m}$	$=$	optimal $\mathbf{s}$ vector (having maximum weight)
W_T	$=$	total amount of work (number of links on which IC is performed) done by all transmitters
W_R	$=$	total amount of work done by all receivers

The optimal throughput is therefore $S_{max} = w(\mathbf{s_m})$. Note that we omit link data rate in the throughput expression. This is because, in this paper, we focus on spatial multiplexing (SM) and spatial reuse (SR) only, while not considering using MIMO for improving spatial diversity (and, hence, perceived SNR values at receivers). If only SM and SR are exploited, the rate on a single stream can be considered as a constant, and optimal throughput is then achieved when the maximum number of streams are successfully activated.

In the following sections, we report details of the following analyses:

1) When only SR is performed, i.e. when each link carries only one data stream, we obtain an analytical expression for the optimal throughput or equivalently the number of links that may be active simultaneously.
2) When only SM is performed, our single collision domain assumption implies that the optimal solution can be trivially computed: only a single link is active, and k streams are sent along the active link, which yields an optimal throughput of k, independently of the number l of links to schedule.
3) When both spatial reuse and spatial multiplexing are performed together, we obtain an analytical expression for the optimal throughput as a function of the number of simultaneously active links.
4) Moreover, at this optimal point, we determine how the work of interference cancellation is distributed among all the transmitters and receivers.

IV. Spatial Reuse Only

With spatial reuse only, each active link supports only one stream. Transmitters can use their remaining DOFs to null the signal at some receivers, and receivers can use their remaining DOFs to suppress interference from some transmitters. Thus, $S_{max} = l$. We now find the maximum number of links that can be active simultaneously without any collisions.

Lemma 1: The number of links that can be active simultaneously without collision is at most $2k-1$.

Proof: Assume $l = 2k-1+x$ for integer $x > 0$. We show by simple counting that there are not enough DOF's to allow l links to be active simultaneously. Let DOF_A be the total number of DOFs available among the l links. Since transmitter and receiver arrays have size k, $DOF_A = l \cdot 2k = (2k-1+x) \cdot 2k = 4k^2 - 2k + 2kx$. Let DOF_R be the total number of DOFs required to allow the l links to be active. Each transmit-array and receive-array uses one DOF to support the single data stream. The interference generated by each transmission must be canceled out for every other link, either by the transmitter or by the other link's receiver. Thus, $DOF_R = 2.(2k-1+x) + (2k-1+x)\cdot(2k-1+x-1) = 4k^2 - 2k + 4kx + x^2 - x$. Since $x > 0$, $DOF_R > DOF_N$ and there is no way for these transmissions to succeed simultaneously. ■

We now show that the upper bound of $2k-1$ simultaneously active links is achievable.

Lemma 2: There is an assignment of MIMO weights that will allow l links to be active simultaneously without collision, if $l \leq 2k-1$.

Proof: : With MIMO links having antenna array size of k, the transmitter on an active link can set its weights so as to null its transmitted signal at an arbitrary set of $k-1$ receivers. Similarly, the receiver on an active link can set its weights so as to suppress interference from an arbitrary set of $k-1$ transmitters.

Case 1: $l \leq$k

In this case, each transmitter can null its signal at every other receiver, thus completely eliminating interference.

Case 2: $k+1 \leq l \leq 2k-1$

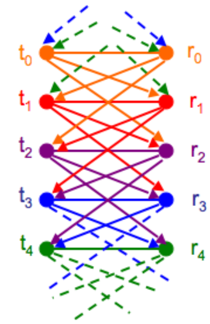

Fig. 1. Optimal Construction with Spatial Reuse Only ($k = 3, l = 5$)

Let $l = k + x$, where $1 \leq x \leq k-1$. All additions and subtractions in the following are taken modulo $(k+x)$. Let the links be denoted by $l_0 = (t_0, r_0), l_1 = (t_1, r_1), \ldots, l_{k+x-1} = (t_{k+x-1}, r_{k+x-1})$. Consider the following use of MIMO weights. Each transmitter t_i sets its weights to null its signal at receivers $r_{i+1}, r_{i+2}, \ldots, r_{i+k-1}$. This uses all remaining DOFs at the transmitters. However, transmitter t_i will still interfere at receivers $r_{i+k}, r_{i+k+1}, ..., r_{i+k+x} = r_{i-1}$. Thus, each of those receivers must use one of their DOFs to suppress the signal from t_i. Assume that each receiver attempts to suppress every transmitter that has not nulled its signal at that receiver. Since $k-1$ transmitters will null their signals at a given receiver and the transmitter on the receiver's link should not be suppressed, this leaves $l-k$ transmitters for the receiver to suppress. The receiver has $k-1$ DOFs to use for suppression. Since $l-k = k+x-k = x \leq k-1$, each receiver has enough DOFs to suppress every remaining interfering transmitter. Therefore, all links can be active simultaneously without collision. ■

An example of the construction of Lemma 2 is shown in Figure 1 for $k = 3$ and $l = 2k-1 = 5$. In this figure, each link (including its transmitter and receiver) is depicted in a different color. Edges depict DOFs being used by transmitters or receivers and are assigned the same color as the node that is using its DOF. Each receiver has 2 different-colored incoming edges, indicating that the transmitters of those colors are using their DOFs to null their signal at that receiver. In addition, each receiver has 2 outgoing edges of its own color, which represent the use of its remaining DOFs to suppress certain transmissions. Finally, note that the outgoing edges from each receiver terminate at exactly the nodes whose colors are not represented by incoming edges to that receiver, i.e. the receiver is suppressing interference from all interfering transmitters that did not null their incoming signals to it.

Theorem 1: The optimal throughput with l MIMO links, each having array size k, in a single collision domain with spatial reuse only is:

$$w(\mathbf{s_m}) = \begin{cases} l & \text{if } l \leq 2k-1 \\ 2k-1 & \text{if } l > 2k-1 \end{cases}$$

Proof: The theorem follows directly from Lemmas 1 and 2 and the fact that, with spatial reuse only, there is one stream per active link. ■

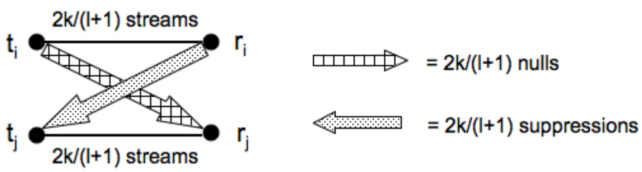

Fig. 2. Interference Cancellation between Two Links l_i and l_j

V. Spatial Reuse and Spatial Multiplexing

When it is possible to use both spatial reuse (SR) and spatial multiplexing (SM) with MIMO links, the optimal way to use the MIMO DOFs is not obvious. The transmitters and receivers of a given link could use their DOFs to multiplex several streams, thereby increasing the link data rate, or they could use their DOFs to cancel interference, thereby allowing more links to be simultaneously active. Of course, one obvious case is when $l = 1$. In this case, there is no interference and the link can use its full multiplexing potential to support k data streams, resulting in a throughput of k. However, it is only about half of the maximum throughput achievable with spatial reuse only, which is $(2k-1)$. Thus, the use of multiple simultaneously active links can increase throughput, even when spatial multiplexing is allowed. However, in this situation, IC will be necessary, meaning that it is not possible to use all DOFs to achieve spatial multiplexing.

A. Lower Bound on Achievable Throughput

The following theorem provides a constructive lower bound on throughput when both SR and SM are performed.

Theorem 2: Assume $1 \le l \le 2k-1$, and let l' be the largest integer not greater than l for which $\frac{2k}{l'+1}$ is an integer. Then there exists an assignment of MIMO weights that can support l'$\frac{2k}{l'+1}$ simultaneously active streams without collisions when spatial reuse and spatial multiplexing are both used.

Proof: Case 1: $\frac{2k}{l+1}$ is an integer

Let the links be denoted by $l_0 = (t_0, r_0), \ldots, l_{l-1} = (t_{l-1}, r_{l-1})$. Assign $\frac{2k}{l+1}$ streams to every link. The total number of streams is $l\frac{2k}{l+1}$, which matches the number in the theorem. Now, consider interference between two arbitrary links l_i and l_j. l_i can eliminate interference between these links (in both directions) by using $\frac{2k}{l+1}$ degrees of freedom at both t_i and r_i. r_i. This situation is depicted in Figure 2. The total number of bi-directional interference cancellations a given link can achieve in this manner is $\frac{k-\frac{2k}{l+1}}{\frac{2k}{l+1}} = \frac{l-1}{2}$. Note that in the case under consideration, l must be odd so that this value is an integer. So, any given link can cancel interference in both directions with exactly $\frac{l-1}{2}$ other links and will completely use its DOFs to do so. Interference must be canceled between all $\binom{l}{2} = l \cdot \frac{l-1}{2}$ pairs of links. Since each link can cancel interference with $\frac{l-1}{2}$ others, the number of possible cancellations matches the number required.

We now give an exact assignment of interference cancellations that will allow these l links to be simultaneously active. All additions and subtractions are modulo l. For any given link l_i, use the DOFs at its transmitter and receiver to cancel interference with $l_{i+1}, l_{i+2}, \ldots, l_{i+\frac{l-1}{2}}$. An example of this assignment for $l = 7$ is shown in Figure 3. Now, consider an arbitrary link l_j , with $j \neq i$. We claim that with the given assignment, interference is cancelled between l_i and l_j . If $j \le i + \frac{l-1}{2}$ then this is true from l_i's DOF assignment. So, consider the case where $j > i + \frac{l-1}{2}$. Then, it must be true that $i \le j + \frac{l-1}{2}$ (recall that l is odd and arithmetic is modulo l). Thus, in this case, l_j will have been assigned to cancel interference with l_i. Since l_i and l_j are arbitrary links, this implies that all pairs of links are covered by this assignment, and all interference is cancelled. This can be seen in the Figure 3 example where each link has exactly one incoming or outgoing edge connecting it to every other link.

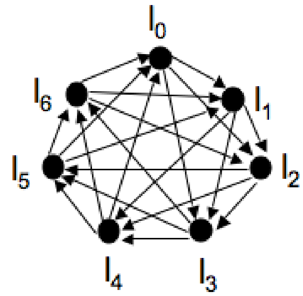

Fig. 3. Interference Cancellation Assignment for $l = 7$

No. of links	1	3	5	7	9	11	13	15
No. of streams(SRO)	1	3	5	7	9	11	13	15
No. of streams (SR+SM)	8	12	12	14	14	14	14	15

TABLE I

No.of active streams with SR only and with SR+SM ($k = 8$)

Case 2: $\frac{2k}{l+1}$ is not an integer

In this case, the theorem states that $l'\frac{2k}{l'+1}$ streams can be simultaneously active, where l' is the largest integer smaller than l for which $\frac{2k}{l'+1}$ is an integer. Here, we can simply apply the Case 1 construction for $l = l'$and leave the remaining $l - l'$ links unused, and the theorem's statement is true. ∎

Note that plugging $l = 2k - 1$ into Theorem 2 yields a number of simultaneously active streams equal to $2k - 1$. This is the same value that results from the construction of Lemma 1 for spatial reuse only. In fact, the construction of Theorem 2 degenerates to spatial reuse only in this limiting case. However, for a smaller number of links, Theorem 2's construction allows for a higher number of active streams than with spatial reuse only in most cases. For example, Table I compares the number of active streams achievable for the two scenarios when $k = 8$.

The next sections provide a matching upper bound to the achievable throughput for odd values of l. The optimal throughput will be shown to be achieved when the work of interference cancellation is equally distributed among all transmitters and receivers.

B. Spatial Multiplexing with IC: A Matrix Formulation

Interference cancellation between $\frac{l(l-1)}{2}$ pairs of links ($l(l-1)$ cancellations total) must be done. Thus,

$$W_T + W_R = l(l-1)$$

For the transmitter side, we have

$$s_i + \sum_{j \in L_i} s_j \leq k$$

where L_i is the set of links at which the transmitter of link i nulls itself. Rewrite this as

$$A_1 \mathbf{s} \leq \mathbf{k} \text{ where } l \leq w(A_1) \leq l^2$$

Similarly, for the receiver side, we have

$$s_i + \sum_{j \in M_i} s_j \leq k$$

M_i is the set of links whose transmissions the receiver of link i suppresses. Rewrite this as

$$A_2 \mathbf{s} \leq \mathbf{k} \text{ where } l \leq w(A_2) \leq l^2$$

Note that $w(A_1) = W_T + l$ and $w(A_2) = W_R + l$. The solution $\mathbf{s}$ must satisfy

$$A_1 \mathbf{s} \leq \mathbf{k} \text{ and } A_2 \mathbf{s} \leq \mathbf{k}$$

The optimal solution is the vector with maximum weight subject to the constraints $A_1 \mathbf{s_m} \leq \mathbf{k}$ and $A_2 \mathbf{s_m} \leq \mathbf{k}$, i.e.

$$\mathbf{s_m} = \max_{A_1, A_2} \{ w(\mathbf{s_m}) \rightarrow \mathbf{s_m} : A_1 \mathbf{s_m} \leq \mathbf{k} \text{ and } A_2 \mathbf{s_m} \leq \mathbf{k} \}$$

A_1 and A_2, however, are related. Any choice of A_1 completely determines A_2 and vise-versa. The relation is

$$\begin{cases} A_2(j,i) = 1 - A_1(i,j) \ \forall i \neq j \\ A_1(i,i) = 1; A_2(i,i) = 1 \ \forall i \end{cases} \tag{1}$$

Equation 1 follows from the fact that if transmitter i nulls itself at receiver j, then receiver j need not suppress the signal from transmitter i (coordinated IC). We therefore have

$$A_2 = I + 1 - A_1^T$$

The relation between the weights of A_1 and A_2 naturally follows from this as

$$w(A_1) + w(A_2) = 2l + l(l-1) = l(l+1) \tag{2}$$

The optimal solution can now be simplified as

$$\mathbf{s_m} = \max_{A_1} \{ w(\mathbf{s_m}) \rightarrow \mathbf{s_m} : A_1 \mathbf{s_m} \leq \mathbf{k}, \ (I+1-A_1^T) \mathbf{s_m} \leq \mathbf{k} \}$$

Now, define the mapping f from two vectors $\mathbf{s_1}$ and $\mathbf{s_2}$ to choose the vector with minimum weight as

$$f : \{\mathbf{s_1}, \mathbf{s_2}\} \rightarrow 1\{w(\mathbf{s_2}) \geq w(\mathbf{s_1})\} \mathbf{s_1} + 1\{w(\mathbf{s_1}) > w(\mathbf{s_2})\} \mathbf{s_2}$$

where the function $1(C) = 1$ if C is true and 0 otherwise. The optimal solution amounts to maximizing the minimum-weight vector of the two vectors $\mathbf{s_1}$, $\mathbf{s_2}$ which satisfy $A_1 \mathbf{s_1} = \mathbf{k}$ and $A_2 \mathbf{s_2} = \mathbf{k}$. This is expressed as

$$\begin{aligned} \mathbf{s_m} = \quad & \max_{A_1} \Big\{ w(f(\mathbf{s_1}, \mathbf{s_2})) \rightarrow f(\mathbf{s_1}, \mathbf{s_2}) : A_1 \mathbf{s_1} = \mathbf{k} \\ & \text{and } (I + 1 - A_1^T) \mathbf{s_2} = \mathbf{k} \Big\} \end{aligned} \tag{3}$$

Equation 3 follows because

$$\begin{aligned} & \text{If } w(\mathbf{s_1}) > w(\mathbf{s_2}) \\ \Longrightarrow \quad & w(A_1) < w(I + 1 - A_1^T) \\ \Longrightarrow \quad & (A_1 + A_1^T) \mathbf{1} < (I+1) \mathbf{1} \\ \Longrightarrow \quad & (A_1 + A_1^T) \mathbf{s_2} < (I+1) \mathbf{s_2} \ (\because \mathbf{s_2} \text{ is positive}) \\ \Longrightarrow \quad & A_1 \mathbf{s_2} - (I + 1 - A_1^T) \mathbf{s_2} < 0 \\ \Longrightarrow \quad & A_1 \mathbf{s_2} < k \end{aligned}$$

(and vice-versa with s_1 and s_2 interchanged.)

Now, $w(A_1) = l \implies A_1 = I^{lxl}$. This represents one extreme where the total work done by transmitters is zero, i.e. $W_T = 0$. All work is done by the receivers, i.e. $W_R = l(l-1)$. In this case, $A_2 = \mathbf{1}$ and $w(A_2) = l^2$. At the other extreme, we have the transmitters creating nulls at every receiver ($w(A_1) = l^2, A_1 = \mathbf{1}, W_T = l(l-1)$) and the receivers doing zero work ($w(A_2) = l, A_2 = I^{lxl}, W_R = 0$).

To find the optimal solution, we evaluate the right hand side of Equation 3 for all A_1 with weight ranging from $w(A_1) = l$ [corresponding to $W_T = 0, W_R = l(l-1)$] to $w(A_1) = l^2$ [corresponding to $W_T = l(l-1), W_R = 0$]. We obtain a set of $l \cdot (l-1) + 1$ solutions, each being maximal over the class of matrices having a certain weight. The optimal solution is the maximum of this set. In practice, we do not need to sweep the weight of A_1 beyond the midpoint ($W_T = W_R = \frac{l(l-1)}{2}$) up to $W_T = l(l-1)$ because of the following property.

Lemma 3: The MIMO system under consideration is equivalent to its dual configuration, obtained by reversing the direction of every communication link.

Proof: We model transmitters and receivers identically, i.e. transmitter and receiver arrays have equal numbers of antenna elements and identical signal processing capabilities. Thus, reversing the roles of transmitters and receivers and the directions of data transfer, preserves the throughput. The roles of A_1 and A_2 are then reversed, i.e. A_1 is the receiver side matrix and A_2 is the transmitter side matrix. ■

C. Lagrange Multiplier Method of Optimization

For every value 'w' of $w(A_1) \in [l, l^2]$, we apply the Lagrange Multiplier Method of optimization to find the maximal solution

$$\begin{aligned} \mathbf{s_m^w} \quad = \quad & \max_{A_1} \Big\{ w(f(\mathbf{s_1}, \mathbf{s_2})) \rightarrow f(\mathbf{s_1}, \mathbf{s_2}) : A_1 \mathbf{s_1} = \mathbf{k} \ , \\ & (I + 1 - A_1^T) \mathbf{s_2} = \mathbf{k} \ , \ w(A_1) = w \Big\} \end{aligned} \tag{4}$$

Finally, the optimal solution is calculated as

$$\mathbf{s_m} = \max_{w(A_1) = w} \{ \mathbf{s_m^w} \} \tag{5}$$

We will see that this maximum occurs when $w(A_1) = \frac{l(l+1)}{2}$. Moreover, Equation 2 gives $w(A_2) = l(l+1) - w(A_1) = w(A_1)$. Let the weights of A_1 and A_2 be

$$\begin{aligned} w(A_1) &= l+n \text{ where } 0 \leq n \leq l \cdot (l-1) \\ \implies w(A_2) &= l^2 - n \end{aligned}$$

We have, for the transmitter side,

$$\begin{aligned} & A_1 \mathbf{s_1} = \mathbf{k} \\ \implies & s_1 c_1^T + s_2 c_2^T + \ldots s_l c_l^T = \mathbf{k} \end{aligned}$$

where c_i is the i^{th} column of A_1 and s_i is the i^{th} element of $\mathbf{s_1}$. Denoting $w(c_i)$ by A_i^w,

$$s_1 A_1^w + s_2 A_2^w + \ldots s_l A_l^w = kl \tag{6}$$

We also have

$$\begin{aligned} & w(A) = l+n \\ \implies & A_1^w + A_2^w + \ldots A_l^w = l+n \end{aligned} \tag{7}$$

We want to maximize the function

$$f(\mathbf{s_1}, \mathbf{A_1}) = s_1 + s_2 + \ldots s_l \tag{8}$$

where $\mathbf{s_1} = (s_1, \ldots s_l)$ and $\mathbf{A_1} = (A_1^w, \ldots A_l^w)$

subject to the following two constraints:

$$\begin{aligned} \phi(\mathbf{s_1}, \mathbf{A_1}) &= s_1 A_1^w + s_2 A_2^w + \ldots s_l A_l^w \\ & \quad -kl = 0 \text{ and} \\ \theta(\mathbf{s_1}, \mathbf{A_1}) &= A_1^w + A_2^w + \ldots A_l^w \\ & \quad -(l+n) = 0 \end{aligned}$$

This is done by the method of Lagrange multipliers as follows. Define

$$\begin{aligned} & F(\mathbf{s_1}, \mathbf{A_1}, \lambda, \mu) = f(\mathbf{s_1}, \mathbf{A_1}) - \lambda \phi(\mathbf{s_1}, \mathbf{A_1}) - \mu \theta(\mathbf{s_1}, \mathbf{A_1}) \\ = & \ s_1 + s_2 + \ldots s_l - \lambda(s_1 A_1^w + \ldots + s_l A_l^w \\ - & \ kl) - \mu(A_1^w + \ldots + A_l^w - (l+n)) \end{aligned}$$

Then, we solve the system

$$\frac{\delta F}{\delta s_i} = 0 \ \forall i = 1 \ldots l \tag{9}$$

$$\frac{\delta F}{\delta A_i^w} = 0 \ \forall i = 1 \ldots l \tag{10}$$

$$\phi(\mathbf{s_1}, \mathbf{A_1}) = 0 \tag{11}$$

$$\theta(\mathbf{s_1}, \mathbf{A_1}) = 0 \tag{12}$$

This gives

$$\begin{aligned} & A_1^w = \ldots = A_l^w = \frac{1}{\lambda} \text{ (from equation 9)} \\ & s_1 = \ldots = s_l = \frac{-\mu}{\lambda} \text{ (from equation 10)} \\ & l \frac{-\mu}{\lambda} \frac{1}{\lambda} = kl \text{ (from equation 11)} \\ & \implies \mu = -k\lambda^2 \\ & l \frac{1}{\lambda} = (l+n) \text{ (from equation 12)} \\ & \implies \lambda = \frac{l}{l+n} \end{aligned}$$

Finally, we have

$$\begin{aligned} & A_1 = \ldots A_l = (\frac{n}{l} + 1) \text{ and} \\ & s_1 = \ldots = s_l = k \cdot \lambda = k \frac{l}{l+n} \end{aligned}$$

And so, the maximum value of the function $f(\mathbf{s_1}, \mathbf{A_1}) = s_1 + s_2 + \ldots + s_l$ is

$$w_m(\mathbf{s_1}, n) = k \frac{l^2}{l+n}$$

Similarly, for the receiver side, we obtain

$$w_m(\mathbf{s_2}, n) = k \frac{l^2}{l^2 - n}$$

Evaluating the optimal solution from equations 4 and 5 amounts to evaluating

$$\begin{aligned} w(\mathbf{s_m}) &= \max_n \Big\{ min\{w_m(\mathbf{s_1}, n), w_m(\mathbf{s_2}, n)\} \Big\} \\ &= k \frac{2l}{(l+1)} \end{aligned}$$

This maximum occurs at $n = \frac{l(l-1)}{2}$. Correspondingly, $\lambda = \frac{2}{l+1}$. Therefore

$$\begin{aligned} w(A_1) &= l + n = \frac{l(l+1)}{2} \\ w(A_2) &= l^2 - n = \frac{l(l+1)}{2} \end{aligned}$$

Hence, optimal throughput $\mathbf{s_m}$ occurs at the mid-point where $w(A_1) = w(A_2)$ i.e. transmitters and receivers share work equally. $S_{max} = w(\mathbf{s_m}) = k\frac{(2l)}{l+1} = \frac{2kl}{l+1}$. For this to be integral, k should be a multiple of $\frac{l+1}{2}$. Our analysis cannot be completed before making the following important observation.

Note: The values of A_i^w should be integral as these are the weights of the columns of A_1, which have '1' and '0' as entries. However, we have disregarded this fact and carried out the Lagrange Multiplier Method of optimization in the Real domain. At the optimal point (mid point), the value of the column weights, $\frac{1}{\lambda} = \frac{l+1}{2}$ is integral if l is odd. Furthermore, optimal values of A_i^w yielded by the Lagrange Method are integral also for those values of n which are multiples of l. For all other values of n, the column weights obtained are non-integer. Given that we have determined that the value of $min\{w_m(\mathbf{s_1}, n), w_m(\mathbf{s_2}, n)\}$ is strictly lower than $k\frac{2l}{l+1}$ for all n other than $n = \frac{l(l-1)}{2}$, imposing the integer constraint would only further strengthen the inequality. Therefore the approach is justified, and we are safe in performing the optimization in the Real domain.

D. Structure of the Matrices at Optimal Point

At the optimal point, we have $w(A_1) = w(A_2)$. Moreover, the outcome of the Lagrange Method gives the weight of each row and of each column of A_1 and A_2 to be equal to $\frac{(l-1)}{2} + 1$. This result translates to our MIMO setting to mean that every transmitter and every receiver performs interference cancellation with $\frac{(l-1)}{2}$ other links. That is to say, the work of IC is equally distributed among all transmitters and receivers.

Finally, the relation $A_2 = I + 1 - A_1^T$ implies $A_1 = A_2$. Hence, the transmit and receive matrices are equal at the optimal point. We can therefore write

$$A_1 = A_2 \implies A_1 + A_1^T = I + 1 \implies \begin{cases} A_1(i,j) = 1 - A_1(j,i) \ \forall i \neq j \\ A_1(i,j) = A_2(i,j) \end{cases} \tag{13}$$

Equation set 13 is the ***Symmetry Condition***. The symmetry condition implies that if transmitter i nulls itself at receiver j, then transmitter j will not null itself at receiver i. Instead, it is receiver i which suppresses the signal from transmitter j. *Therefore, interference cancellation between a pair of links is done entirely by one of the links at the optimal point.*

E. Handling Primary Interference

Primary interference occurs when a station is involved in more than one communication task at the same time (sending and receiving, receiving from two different transmitters, etc.). Let $G = (V, E)$ be a subgraph of the communication graph of the system containing all links that have data to transmit. In general, the links of G are *not* free of primary interference.

A matching of G is a set of edges, where each vertex appears in at most one edge of the matching. Thus, by definition the links making up a matching of G *are* free of primary interference. The following theorem demonstrates that, if we obtain a maximum matching M of G and apply our optimal construction from the previous sections to the links contained in M, then this achieves the maximum throughput possible among all of the links in G. This then provides an optimal solution to the one-shot stream scheduling problem under consideration.

Theorem 3: Let M be a maximum matching among all links having data to transmit. Let the number of links in *M* be $l \in [1, 2k-1]$ such that $\frac{2k}{l+1} \in Z$. This implies $l = 2m+1$ for some $m \in N$ and k is a multiple of $(m+1)$. Then the optimal throughput is $S_{max} = \frac{2kl}{l+1}$.

Proof: In subsections B and C, it was proven that in the absence of primary interference, the optimal throughput supported by a set of $l \leq 2k-1$ links in a single collision domain is $S_{max} = \frac{2kl}{l+1}$ for odd l. Since this is an increasing function of l, no set of links which form a matching smaller than l (size of the maximum matching) can support a higher throughput. ■

Note that Theorem 3 holds only if the number of links in the maximum matching is odd. Exactly characterizing the optimal solution for an even number of links is an open problem, which we are considering in future work. However, the optimal constructions for odd numbers of links can be used to achieve bounds on optimality for even numbers of links. For example, in Table I, observe that for 6 links, we can achieve 12 streams using the optimal construction for 5 links. Furthermore, the optimal for 6 links can be no better than the optimal for 7 links, which achieves 14 streams. Therefore, the 5-link construction is within two streams of optimal for 6 links. Note also that as the number of links increases, this bound gets tighter. From the same table, we can see that the optimal construction for 7 links is also guaranteed to be optimal for 8, 10, and 12 links. In general, if we use the optimal construction for $l-1$ links for an even l, we can get within at least $\frac{4k}{l(l+2)}$ of optimal, which decreases in proportion to l^2.

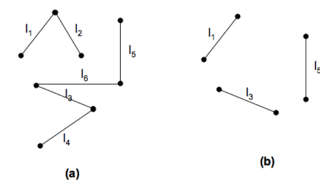

Fig. 4. (a) Communication graph in a single collision domain (b) Maximum matching of the communication graph

n	0	1	2	3	4	5	6
$w_m(\mathbf{s_1}, \mathbf{n})$	6	4.5	3.60	3	2.5714	2.25	2
$w_m(\mathbf{s_2}, \mathbf{n})$	2	2.25	2.5714	3	3.60	4.5	6
$w(\mathbf{s_m})$	2	2.25	2.5714	3	2.5714	2.25	2

TABLE II

OUTPUT OF THE LAGRANGE MULTIPLIER OPTIMIZATION METHOD.

F. An Example

Consider a single collision domain with 6 links as shown in Figure 4a. Assume all links have data to transmit. We want to schedule the maximum number of streams possible in one time slot across these links. Obtain a maximum matching of size 3 as shown in Figure 4b. Thus we get the maximum number of primary-interference-free links to be $l = 3$. Choose the size of the antenna array to be $k = 2$. We will apply the optimization procedure derived above to this setting in order to find the maximal throughput and the structure of the A_1 and A_2 matrices at the optimal point.

The weight of A_1 is swept from $n = 0$ to $n = 6$. Note the symmetry about the mid-point, which is a result of the duality property. For values of n smaller than the mid-point i.e. for $n < 3$, we have $w(A_1) < w(A_2)$. This implies $W_T < W_R$ i.e. the total work done by the transmitters in IC is less than that done by the receivers. The number of degrees of freedom available at the transmitters for multiplexing data streams is therefore larger than that available at the receivers. Hence the receiver side matrix A_2 determines (imposes a stronger constraint on) the achievable throughput. This is evident from Table II. On the other hand, for values of n larger than the mid-point i.e. for $n > 3$, we have $w(A_1) > w(A_2)$. In this case, the achievable throughput is constrained more strongly by the transmit side matrix A_1 as seen in the table.

VI. DISCUSSION

In this section, we qualitatively compare our results with the ones (based on numerical evaluation) reported in [6]. We stress that the network setting considered in [6] is quite different from ours: multi-hop flows are to be scheduled on a set of links with arbitrary collision domains. On the other hand, our approach assume single links to be scheduled (one-hop flows), and all links are part of a single collision domain. Despite the

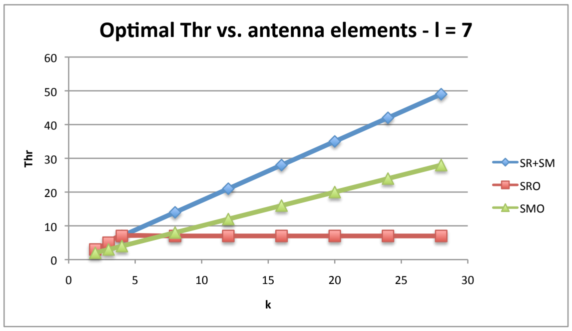

Fig. 5. Optimal throughput achievable with $l = 7$ links and increasing number of antenna elements, with SR only, SM only, and SM+SR MIMO systems.

different network settings, the main qualitative findings of [6] are fully confirmed by our analytical results. To be specific, Hamdoui and Shin observe that, as the number of antenna element increases, the maximum achievable throughput first raises and then flattens out asymptotically under SRO, while it increases "almost" linearly under SMO or SR+SM. As seen from Figure 5, this behavior can be observed also when the results derived in this paper are extended to arbitrarily large values of k: in case of SRO, the throughput increases when relatively few DOFs are available; once the available DOFs are sufficient to null/suppress all interference, the optimal throughput flattens to the optimal value of l, corresponding to scheduling one stream on each possible link. In case of SMO, only one link can be active at a time. Hence, optimal throughput increases linearly with k, which corresponds to the maximum possible number of streams that can be transmitted on the active link. In case of SM+SR, additional throughput benefit (near two-fold) can be achieved by combining the two MIMO techniques. It is also worth observing that when relatively few DOFs are available ($k \leq 5$), all DOFs are used to mitigate interference (SRO and SM+SR curves are overlapped); as the number of available DOFs increases, interference mitigation can be combined with spatial multiplexing to achieve considerable throughput gains over the SRO and SMO approaches.

VII. Conclusions

For spatial reuse only, we obtained an expression for the maximum throughput of a single-hop network as a function of the number of links and antenna array size. For spatial reuse and spatial multiplexing, we derived algebraically an expression for the maximum throughput when the number of links is odd. We showed that optimum throughput is achieved when the work of interference cancellation is shared equally between every transmitter and every receiver, and therefore all links multiplex the same number of streams. Moreover, we showed that at the optimum point, the interference between every pair of links is canceled entirely by one of the links. That is, if the transmitter of a given link nulls itself at the receiver of an interfering link, then the receiver of the given link will suppress the signal from the transmitter of the interfering link. Finally, we showed that the optimal throughput obtained with spatial multiplexing and spatial reuse combined ($S_{max} = \frac{2kl}{l+1} \approx 2k$ streams) is approximately twice what it is with spatial multiplexing only (single link with k streams).

VIII. Future Work

We would like to derive the optimum result for an even number of links. The Lagrange multiplier method of optimization must be done in the integer domain rather than the Real domain in this case. We also plan to extend our formulation to study the throughput characteristics of multi-hop networks.

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