Topology Control with Better Radio Models: Implications for Energy and Multi-Hop Interference

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Abstract

Topology Control (TC) is a well-studied technique used in wireless ad hoc networks to find energy-efficient and/or low-interference subgraphs of the maxpower communication graph. However, existing work has the following limitations: (1) the energy model adopted is quite unrealistic - only transmit power is often considered and homogeneous decay of the radio signal with distance is assumed; (2) the interference measure does not account for multi-hop communications. In this paper, we show the dramatic effect of the underlying energy and interference model on TC. In particular, we demonstrate that by using more realistic energy models and considering the effects of multi-hop interference, radically different conclusions about TC can be drawn; namely that (1) energy efficient TC is essentially meaningless, since every link turns out to be "efficient", and that (2) topologies identified as "interference-optimal" in the current literature can be extremely bad from the viewpoint of multi-hop interference. Given these observations, we propose a new measure of link interference, extend it to deal with multi-hop interference, and design a corresponding optimal communication subgraph, called ATASP. We prove that, in the worst case, ATASP coincides with the maxpower communication graph, showing that in some unfortunate situations also performing multi-hop interference-based TC is pointless. However, the simulation results with random node deployments presented in this paper show that, on the average, ATASP is a sparse subgraph of the maxpower communication graph, and multi-hop interference-based TC is indeed possible. Since computing ATASP requires global knowledge, we experiment through simulation with known localized algorithms for energy-efficient TC and show that they perform well (on the average) with respect to multi-hop interference.

Keywords: Ad hoc wireless networks, topology control, radio models, interference, energy consumption.

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1 Introduction

Topology Control (TC) attempts to find efficient but sparse subgraphs of the maxpower communication graph (from now on, maxpower graph) in a wireless ad hoc network [1, 2, 10, 13, 15, 17]. The goal of TC is to eliminate inefficient links that ought not be used for communication. In addition to TC's inherent benefits, use of a sparse topology reduces routing overhead, which can be quite high in ad hoc networks due to expensive flooding of route discovery messages [9].

The efficiency metrics used to date in the TC literature are: (1) energy [1, 10, 15, 17] and (2) interference [2, 13]. The need to reduce energy is fundamental in energy-constrained environments, while reducing interference has the potential to increase network capacity [6, 7, 8].

However, existing work has made some significant simplifying assumptions. First, when considering energy, only transmit power is typically considered and it is assumed that power decays as $\frac{1}{d^\alpha}$, where d is the distance between sender and receiver and α is the path loss exponent. This is known to be a poor model for energy consumption of the entire network interface (as we demonstrate in Section 2). Also when the receiver power is accounted for (as in [15]), the assumption of homogeneous power decay with distance is used, implying that the transmit power varies from nearly 0 (when the receiver is very close to the sender) to high values (when the receiver is far away). Actually, as we discuss in Section 2, in real wireless transceivers the ratio between the minimum and the maximum possible transmit power is limited, and it is often well within a factor 2. As we will discuss therein, accounting for the actual ratio between the minimum and the maximum possible transmit power leads to draw radically different conclusions about which links are energy-efficient.

Simplifying assumptions have been made also when considering interference, namely that (1) the transmission regions are perfectly circular and (2) interference in multi-hop communications is not accounted for.

In this paper, we study the TC problem using more realistic energy and interference models, and we show that if such models are used, radically different conclusions about TC are drawn.

Concerning energy, we show that, at least with current transceiver technology, *no energy-efficient TC is possible*: every link in the maxpower graph is energy-efficient. This statement is first theoretically proved for the case of three nodes (reversing the well-known triangular inequality argument), and then validated through simulation for larger network sizes.

We then move to considering TC for interference. We first point at some limitations of the proposed definitions of (graph) interference and introduce two new metrics, (a) to measure the interference associated to a link in a way that does not depend on assumptions on the radio coverage area, and (b) to account for interference in multi-hop communications. We also present distributed protocols for estimating the link interference in a maxpower graph, first for homogeneous networks and then for networks in which the radio devices are not constrained to use the same power levels. Using our notion of multi-hop interference, we then show that: (i) MST-based topologies (proposed as optimal solutions in current literature [2, 13]) are actually $\Omega(n)$ away from the optimal solution (n is the number of network nodes) if multi-hop interference is accounted for; and (*ii*) there exist node placements and transmit power settings such that removing any link from the maxpower graph results in increasing multi-hop interference.

In light of (*ii*), one might conclude that no multi-hop interference TC is possible as well. However, (*ii*) holds in a worst-case scenario. Is some type of TC possible for non-pathological node placements? To answer this question, we extend an existing interference metric to account for irregular radio propagation and multi-hop interference. Based on this metric, we propose a new network topology, called ATASP, which is shown to be optimal from the point of view of multi-hop interference (i.e., it maintains all the interference-efficient links), and we investigate the properties of this topology through simulation with random node deployments. The results of our simulations show that, if we exclude pathological node placements, multi-hop interference-based TC is actually possible, since most of the links in the maxpower graph can be removed without increasing multi-hop interference.

Unfortunately, computing ATASP requires global knowledge. While we leave the problem of designing a localized TC protocol for building a provably multi-hop interference optimal topology open, we show through simulation that some of the localized protocols proposed for energy-efficient TC actually perform quite well (on the average) with respect to multi-hop interference.

We believe the main contribution of this paper is to make it clear the dramatic impact of the underlying energy and interference model used on the conclusions that can be drawn about the network topology. While it was well known in the community that using "simple" models could lead to "inaccurate" conclusions about the optimal network topology, no research has thoroughly investigated the relations between radio models and the resulting optimal network topology. The results presented in this paper clearly demonstrate the importance of choosing a realistic (although necessarily simplified) radio model when studying fundamental properties of wireless ad hoc networks.

The rest of this paper is organized as follows. In Section 2, we discuss the implications on TC of using more realistic energy models. In Section 3, we consider interference-based TC, and in Section 4 we propose a metric to estimate interference in ad hoc networks. In Section 5, we introduce efficient distributed protocols for computing this metric, under different assumptions about the features of the underlying ad hoc network. In Section 6, we show that existing solutions for interference-based TC are very inefficient when multi-hop interference is accounted for. In Section 7, we present ATASP, a provably optimal topology for multi-hop, interference-based TC, and we analyze its properties. In Section 8, we evaluate through simulation the performance with respect to multi-hop interference of existing localized TC protocols, which have been designed for a different optimization goal (reducing energy consumption). In Section 9, we revisit the wellknown triangular inequality argument in view of reducing multi-hop interference. Finally, Section 10, we draw some conclusions and point to possible direction for future work.

2 TC for energy

In this section, we use the following notation for the power consumption parameters of a network interface: $t_{\max}$ = transmit power at highest setting; $t_{\min}$ = transmit power at lowest setting; r = receive power.

Consider Figure 1, which shows a typical situation involving three nodes having a triangle relationship.

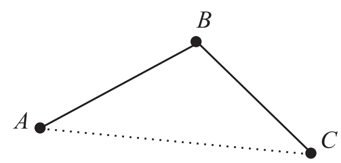

Figure 1: Example of a Triangle Relationship

In this situation, we are concerned with sending data from node A to node C and we would like to decide whether it is more energy-efficient to use the direct link connecting the two nodes or to use the multi-hop path with two links (A, B) and (B, C). We are primarily interested in the question of whether it is ever more energy-efficient (with realistic network interface parameters) to use the multi-hop path. Accordingly, we consider the best-case scenario for the multi-hop path, i.e., the situation where links (A, B) and (B, C) use $t_{\min}$ and link (A, C) uses $t_{\max}$.

Considering both transmit and receive powers, the total power for the single-hop path is $t_{\max} + r$ and the total power for the multi-hop path is $2t_{\min} + 2r$. Thus, the multi-hop path is preferable if and only if:

$$t_{\min} < \frac{t_{\max} - r}{2} \tag{1}$$

Much of the existing work assumes $t_{\max} \gg t_{\min}, r$. In this situation, Inequality 1 would hold. However, this assumption accounts only for the power consumed by the power amplifier but not the *total* power consumed by the interface.

For current technology, and when total power consumption is considered, triangle inequality is hardly satisfied. Data from all network interfaces that we have seen show that r, $t_{\min}$, and $t_{\max}$ are all within a factor of two. Triangle Inequality (1) clearly does not hold for values in this range. For example, in the Cisco Aironet 4800 card, $r = 0.958t_{\min}$ and $t_{\max} = 1.358t_{\min}$ [4]. For Inequality (1), these values make the left hand side 2.5 times greater than the right hand side. In the sensor domain, the Medusa II sensor nodes have $r = 1.107t_{\min}$ and $t_{\max} = 1.265t_{\min}$ [14]. Here, the situation is even less favorable for the multi-hop path in that the left hand side becomes almost 13 times as large as the right hand side!

These simple analyses have a serious implication on topology control for energy reduction. *Because the most energy-efficient path between two nodes that are the endpoints of a wireless link is the link itself, no link is unnecessary if minimum-energy paths are to be used at all times and thus, no topology control is possible*[1].

[1]Note that this statements holds with current transceiver technology, and it might no longer hold when the technology will allow to have t_{min} order of magnitudes lower than t_{max}.

These analyses leave open the theoretical possibility that a path connecting two nodes of length $k+1$ is more energy-efficient than a path between the same nodes of length k, for large enough k. However, in the following simulation results (with the Cisco Aironet 4800 power values), this situation never occurred. The minimum-energy path between two nodes *always* corresponded to a minimum-hop path. This implies that, from a practical standpoint, *energy-aware routing corresponds to selecting the minimum-energy path from among all minimum-hop paths.*

The simulations have been performed considering n nodes (n ranges from 10 to 500) deployed uniformly at random in the unit square. Two radio channel models have been considered: free space propagation (circular radio coverage, with path loss exponent $\alpha = 2$), and log-normal shadowing. In the log-normal shadowing model, the transmitted signal attenuation at a certain distance is determined by the sum of a deterministic and a random component. This way, the log-normal shadowing model accounts for the situations in which the radio coverage area is irregular. The deterministic component gives the average value of the received signal, which is determined by the distance between sender and receiver and by the path loss exponent (set to 2 in our experiments). The random component has log-normal distribution (normal distribution when measured in dBs) with standard deviation σ ($\sigma = 6$ in our experiments).

Note that the log-normal shadowing model defines a *virtual distance* between two nodes, which results from the combination of the deterministic and the random components of the signal attenuation. We can say that two nodes in the log-normal shadowing model are neighbors if and only if their *virtual distance* is below the maximum transmitting range.

The nodes maximum transmitting range in our simulations was set to the value of the critical transmitting range for connectivity, augmented by 50% (see [16]).

The energy cost of link (u, v) is computed according to the following formula:

$$EC(u,v) = E_r + E_{txMin} + (E_{txMax} - E_{txMin}) \left(\frac{dist(u,v)}{Tr} \right)^{\alpha} ,$$

where $dist(u, v)$ is the distance (in case of log-normal shadowing, the virtual distance) between nodes u and v, Tr is the maximum transmitting range, E_r is the energy consumed in receiving a packet, and E_{txMin} and E_{txMax} are the energy consumed at minimum and maximum transmit power, respectively. The values of E_r, E_{txMin} and E_{txMax} are taken from [4].

To evaluate the effect of node concentration on the minimum-energy paths, we have repeated the simulations using the two-dimensional Normal distribution to deploy nodes. Indeed, we have considered only the nodes which are deployed in the unit square: that is, to generate a network with n nodes, we distribute nodes according to the two-dimensional Normal distribution, discarding the node if it falls outside the unit square. In general, we thus need the generation of $n_1 > n$ nodes to build a network with n nodes.

As anticipated above, in all the simulated scenarios, the minimum-energy path between two nodes *always* corresponded to a minimum-hop path.

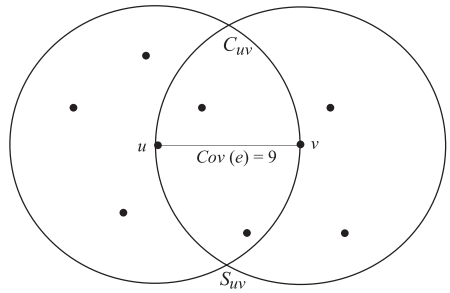

Figure 2: Coverage measure of the edge $e = (u, v)$.

3 TC for interference

The first paper that explicitly addresses the problem of interference-based topology control is [2]. In this work, Burkhart et al. define a metric that estimates the possible interference by a communication along the link. They call this measure *coverage*, which is formally defined as follows:

Definition 1. *Let $e = (u, v)$ be any edge of the maxpower graph $G = (N, E)$, indicating that nodes $u, v \in N$ are within each other's maximum transmitting range. The coverage of edge e is defined as*

$$Cov(e) = |\{w \in N : w \text{ is inside } D(u, \delta(u, v))\} \cup \{w \in N : w \text{ is inside } D(v, \delta(u, v))\}| \ ,$$

where $D(x, y)$ denotes the disk of radius y centered at node x, and $\delta(x, y)$ is the distance between x and y.

The example reported in Figure 2 clarifies the definition of edge coverage.

Based on the notion of link coverage, Burkhart et al. define the *interference* of a certain maxpower graph $G = (N, E)$ as the maximum coverage over all possible links. Formally, $I(G) = \max_{e \in E} Cov(e)$.

Given this notion of graph interference, the authors of [2] identify a set of sparse, connected topologies that minimize interference.

Other interference measures have been proposed in [13]. In particular, Moaveni-Nejad and Li define the interference of a graph as the *average* of the link coverage. Formally,

$$AI(G) = \frac{\sum_{e \in E} Cov(e)}{|E|} \ .$$

We observe that the notions of graph interference introduced in the literature so far suffer two major problems: (i) they are based on the notion of link coverage, which is purely geometric; and (ii) they do not account for interference in multi-hop communications.

Problem (i) implies that the link coverage is an accurate measure of the expected interference only under particular circumstances, i.e., when the radio coverage area can be modeled as a perfect circle. Unfortunately, this is not the case in most practical situations, due to shadowing and fading effects.

Problem (*ii*) can be even more serious, since most communications in ad hoc networks are expected to occur along multi-hop paths. As we shall see, not accounting for multi-hop interference might lead to radically different conclusions about which is the "interference-optimal" topology.

In the next sections, we propose solutions to address these two problems.

4 The interference number

In this section, we introduce a new link metric for estimating interference, which is a generalization of coverage, and we propose a metric to measure interference in multi-hop communications.

As observed in the previous section, the definition of coverage is purely geometric, and it relies on the assumption of perfect circular coverage of the radio signal. That is, this definition relies on a specific radio channel model, which does not account for shadowing and fading effects. Other notions of interference have been recently proposed in [12] and in [13], but they are similar to coverage in that they also are purely geometric definitions and rely on a specific radio channel model.

To circumvent this problem, we generalize the definition of coverage introduced in [2], obtaining a new measure of the interference associated with a link. The most notable aspect of this definition is that *it does not rely on the strong and often unrealistic assumption that the radio coverage area is a perfect circle.* Thus, it can be used in combination with more general radio channel models, which account for shadowing/fading effects.

Definition 2 (Interference number)**.** *Let $e = (u, v)$ be any edge of the maxpower graph $G = (N, E)$, indicating that nodes $u, v \in N$ are within each other's maximum transmitting range. Let $P_u(v)$ (respectively, $P_v(u)$) be the minimum transmit power of node u (respectively, v) needed to sustain the link to node v (respectively, u). Furthermore, let $N_u(v)$ (resp., $N_v(u)$) be the set of nodes within u's (resp., v's) transmitting range when u (resp., v) transmits with power $P_u(v)$ (resp., $P_v(u)$). The interference number of edge e is defined as $IN(e) = |N_u(v) \cup N_v(u)|$.*

The example reported in Figure 3 clarifies the definition of interference number of an edge. We believe the notion of interference number as defined in this paper is a reasonable measure of the interference generated by the communication along a certain wireless link, at least when the MAC layer is based on CSMA-CA (as it is the case of 802.11). Suppose nodes u and v are the communicating nodes; due to the RTS/CTS message exchange, all the nodes within u's and v's transmitting range (i.e., nodes in $N_u(v)$ and in $N_v(u)$) must refrain their communications to avoid interference with the current transmission. So, the number of nodes in $N_u(v) \cup N_v(u)$ (excluding the communicating nodes u and v) is a measure of the amount of wireless medium 'consumed' by the communication.

Note that our notion of interference number can be easily extended to account for interference ranges which are larger than the communication range. This is the case, for instance, when the access to the channel is regulated by a carrier sensing mechanism, given that the carrier sensing range is usually larger than the actual

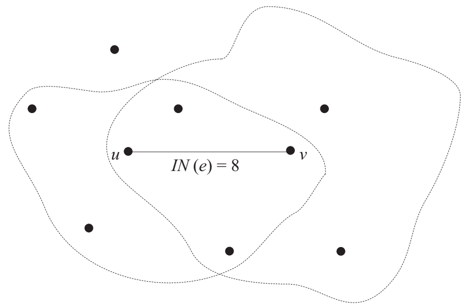

Figure 3: Interference number of the edge $e = (u, v)$.

transmitting range. However, to ease the presentation of our results, in the rest of this paper we assume that the interference range of a node coincides with its transmitting range.

Based on the interference number, we measure interference in a multi-hop communication as follows: given a certain path $p = \{u = w_0, w_1, \ldots, w_{h-1}, w_h = v\}$ connecting nodes u and v, the cost of communicating from u to v along p equals the sum of the interference numbers of the links traversed by the path. This defines the *path interference cost* of p. Formally:

$$PIC(p) = \sum_{i=0}^{h-1} IN((w_i, w_{i+1})) \ .$$

Given the maxpower graph $G = (N, E)$ and a given source/destination pair (u, v) in G, the *minimum interference path* between u and v is a path in G with minimum PIC, and it is denoted $mip^G_{u,v}$.

Based on the PIC, we can use the notion of spanning factor to estimate how good a certain network topology is at reducing interference:

Definition 3 (PIC spanning factor)**.**
Let $G = (N, E)$ be the maxpower graph, and let $G' = (N, E')$ be a subgraph of G. The PIC spanning factor of G' is the maximum over all possible source/destination pairs of the ratio of the cost of a minimum interference path in G' to the cost of a minimum interference path in G. Formally,

$$\rho(G') = \max_{u,v \in N} \frac{PIC(mip^{G'}_{u,v})}{PIC(mip^{G}_{u,v})} \ .$$

Conventionally, we define $\rho(G') = \infty$ if there exist nodes u, v which are connected in G, but they are disconnected in G'.

Ideally, we want to identify a sparse subgraph G' of G with low PIC spanning factor, possibly equal to 1. If such a subgraph G' exists, we are ensured that routing messages along G' does not incur any interference penalty with respect to routing messages in the original graph. Of course, this is true under the assumption that interference-aware routing is used in combination with interference-based topology control.

With respect to this last point, we observe that the PIC can be used to implement interference-aware routing in a straightforward manner, e.g., by using the interference number as the link cost in DSR-like routing protocols [9].

We want to stress that recent work has shown that interference-aware routing has the potential to considerably increase network throughput with respect to shortest path routing (see, e.g., [3] and [8]).

The path interference cost as defined here is the first metric proposed in the literature which: (*i*) can be easily computed; (*ii*) does not require any global knowledge (e.g., number of nodes in the network) nor traffic information; (*iii*) accounts for multi-hop communications; and (*iv*) can be used in combination with transmit power control techniques.

In particular, as already pointed out, the metrics proposed in [2, 13] *do not account for the multi-hop nature of communications in ad hoc networks*. As we will show in the Section 6, not accounting for multihop interference leads to drawing radically different conclusions on which topologies are good for reducing interference.

Other interference metrics have been introduced in the literature. However, they either require knowledge of the traffic flows [12], or they rely on global information such as node positions, density, and expected traffic [7], or they are based on a centralized approach to the problem of reducing multi-hop interference [8]. In the context of routing, the metric that shares most properties with the path interference cost defined here is the expected transmission count metric (ETX) proposed in [3]. ETX estimates the number of transmissions required to successfully deliver a packet over a link, and it is used to find paths that minimize the expected total number of packet transmissions required to successfully deliver a packet to the final destination. Similarly to the PIC metric, ETX can be easily computed relying only on local information (link loss estimate), it does not require global information, and it accounts for multi-hop communications. Furthermore, since the number of expected transmissions is clearly related to the expected interference level in the network, ETX-based routing is likely to select low-interference paths. However, ETX relies on the assumption that all the nodes use a fixed transmit power level, and, consequently, it cannot be used in combination with topology control techniques.

5 The InterfEst protocol

In this section we present distributed protocols for estimating the interference number of the edges in the max-power graph. In particular, at the end of the protocol execution each node in the network knows the interference number of all the edges incident into it. Note that we are interested only in bi-directional links. In other words, we want to estimate the interference number of all the edges (u, v) such that u and v are within each other maximum transmitting range. Our interest in bi-directional links is motivated by the fact that managing unidirectional links incurs a high overhead in ad hoc networks [11].

In the protocol description, we assume that each node can transmit using a finite number of different transmit power levels $P_1, \dots, P_{max}$, which is the case for commercial wireless cards and sensors. Note that *we do not assume that all the nodes use the same power levels, nor that they have the same maximum transmit power*. Thus, our protocol can be applied in networks composed by different types of devices, as is the case in many ad hoc network application scenarios. Furthermore, we do not assume that the wireless medium is symmetric[2].

If all the nodes use the same power levels, and under the assumption of symmetric wireless medium, the protocol definition is considerably simplified. For this reason, we first introduce this simplified version of the protocol, which we call HInterfEst, where H stands for homogeneous networks. Note that wireless sensor networks are typically homogeneous networks.

Protocol HInterfEst:
(protocol for node u)

N_u^i denotes the set of incoming neighbors of node u at power level P_i;

0. *Initialization*
    for i=1 to max do $N_u^i = \emptyset$
1a. *Send beacons*
    for i=1 to max
        send beacon (u, P_i) at power level P_i
1b. *Receive beacon*
    upon receiving beacon message (v, P_j)
        if this is the first beacon message received from v
            $N_u^j = N_u^j \cup \{v\}$
2. *Wait for the stabilization period*
3a. *Send neighbor lists*
    send message $(u, N_u^1, \ldots, N_u^{max})$ at power level P_{max}
3b. *Receive neighbor lists*
    upon receiving message $(v, N_v^1, \ldots, N_v^{max})$
        store the neighbor lists of node v
4. *Wait for the stabilization period*
5. *Compute interference numbers*
    for $i = 1$ to max
        for each $v \in N_u^i$ do
            $IN(e = (u, v)) = |\bigcup_{h=1}^{i} \left(N_u^h \cup N_v^h\right)|$

Figure 4: Protocol for estimating the interference number in homogeneous networks.

5.1 Homogeneous networks

Protocol HInterfEst relies on the following assumptions:

a1. all the nodes can transmit using a finite number of power levels $P_1, \ldots, P_{max}$; the power levels (and, in particular, the maximum transmit power) are the same for all the nodes in the network;

a2. the wireless medium is symmetric.

The protocol is very simple. Initially, every node in the network sends a beacon message at every power level, starting from the lowest level. The beacon contains the node ID and the power level used to send the message.

[2] The wireless medium is symmetric if the fact that node u can reach node v using a certain power P_i implies that also node v can reach node u using power P_i.

When a certain node u receives a beacon message (v, P_i) from a neighboring node, it checks whether this is the first beacon received from v. If yes, node v is inserted in the set of u's incoming neighbors at level i. Otherwise the message is ignored. After a certain stabilization time, needed to receive all possible beacons from the neighbors, node u transmits at maximum power a message that contains its incoming neighbor lists (one for each power level). This information is essential for the nodes in u's neighborhood to compute the interference number of their links to node u. At the same time, node u receives the same type of information from all the incoming neighbors, so that it can compute the interference number of its incident edges. A more formal description of the protocol is reported in Figure 4.

Theorem 1. *Protocol* HInterfEst *is correct, i.e. after its execution every node in the network knows the interference number of every edge incident into it.*

Proof. The correctness of the protocol relies on the following observations, which are a consequence of assumptions *a1.* and *a2.*: when node u receives a beacon message from node v for the first time, the power level used to send that message represents the minimum power needed to sustain the link $e = (u, v)$. Also, due to assumptions *a1.* and *a2.*, the following holds: $v \in N_u^i$ implies $u \in N_v^i$. So, for each power level P_i the nodes in $\bigcup_{h=1}^{i} N_u^h$ are all the nodes within u's range when it transmits with power P_i. Combining these observations, we have that the interference number of link $e = (u, v)$ is given by the union of all the neighbor lists of nodes u and v up to level i, where P_i is the minimum power needed to sustain link e.
To end the proof of the theorem, we need to show that, after HInterfEst execution, u knows the interference number of all the edges incident into it. This follows immediately by observing that, by assumptions *a1.* and *a2.*, every node within u's maximum transmitting range is included in at least one of the N_u^i lists. □

Theorem 2. *Protocol* HInterfEst *has* $O(n(max + 1))$ *message complexity (where* n *is the number of nodes in the network).*

Proof. The proof is immediate, since every node in the network sends one beacon for each power level (max messages overall), plus the final message containing its neighbor lists. □

Theorem 3. *Let* $G = (N, E)$ *be the maxpower graph, and assume* Δ *is the maximum node degree in* G. *Then,* HInterfEst *is at most a factor* $O(\Delta max)$ *away from the message optimal distributed protocol* $\mathcal{P}$ *for computing the interference number of all the links in* G. *This is true under the assumption that* $\mathcal{P}$ *cannot rely on a specific radio channel model to estimate the power needed to sustain a link and that* G *is connected.*

Proof. Let u and v be any two nodes that are adjacent in the maxpower graph. We consider two almost identical scenarios, in which the only difference is the presence or absence of, e.g., an artificial obstacle between u and v. When the obstacle is removed, the link (u, v) can be sustained at power P_i, otherwise power P_{i+1} is required. Suppose the obstacle is removed; we claim that, according to the optimal protocol $\mathcal{P}$, at least one among u and v will eventually send a message at power P_i during the protocol execution. Suppose, for the sake of contradiction, that none of them sends such a message. Then one of the following will occur: (1) u will not hear any message from v and vice-versa, since both nodes only send messages at lower power than P_i, if any; (2) u (or v) will receive a message from v (resp., u) at power at least P_{i+1}. In both cases, it is impossible for u and v to tell the correct power level needed to substain (u, v), because messages at power $p < P_i$ or $p \geq P_{i+1}$ cannot discriminate between the two proposed scenarios.

From the argument above, we can then conclude (without any reference to the power levels) that, for any edge in G, at least one of its endpoints will send a message during the execution of the optimal protocol $\mathcal{P}$. Now, since G is connected, the edge set E contains at least $n-1$ elements. Thus, the bound Δ on the node degree allows us to conclude that there are at least $(n-1)/\Delta$ vertex disjoint edges in G (analogously, we can say that there is a matching of size at least $(n-1)/\Delta$ in G) and this implies that at least $O(n/\Delta)$ messages will circulate under $\mathcal{P}$. In turn, this means that the message complexity of our HInterfEst protocol is within $O(\Delta max)$ from optimal. □

As an additional remark, we note that, unless (u, v) can be sustained at minimum power, at least *two* messages are needed to detect the actual power P_i required (i.e., one at power P_{i-1} and one at power P_i). Thus, unless the majority of nodes can be sustained at minimum power, we obtain a slightly better constant term in the approximation factor even for worst-case graphs.

In words, Theorem 3 states that our algorithm exchanges at most a factor of $O(\Delta max)$ more messages than the minimum required to compute the interference number of all the links in the graph. Although Δ can be as high as $n-1$ in worst-case scenarios, it is known that $\Delta \in O(\log n)$ in most situations [5, 16]. So, our algorithm provides a good approximation of the message optimal solution in most practical cases.

5.2 Heterogeneous networks

We now consider the more complex case of networks composed by heterogeneous devices, i.e., we drop assumption *a1*. We also drop the assumption of symmetric wireless medium. For the sake of presentation only, we assume that all the nodes have the same *number* of power levels, which we denote $P_1, \ldots, P_{max}$. We remark that, contrary to case of homogeneous networks, power level P_i of node u can be different from the same level P_i of another node v. So, P_i here denotes the i-th power level of the nodes, rather than the actual power used to transmit a message.

The protocol InterfEst for heterogeneous networks is more complex than HInterfEst, since in this case $v \in N_u^i$ does not imply that $u \in N_v^i$. Also, the power level needed for node u to sustain the link to node v might be different from the level needed by node v. In order to circumvent these problems, each node now maintains two sets of neighbor lists: the lists of its incoming neighbors (one list for each level), denoted $N_u^{IN,i}$, and the lists of its outgoing neighbors (one list for each level), denoted $N_u^{OUT,i}$. The first set of lists is built as long as the beacons sent by neighboring nodes are received, as in the HInterfEst protocol. Once the incoming neighbor lists are built, they are included in the incoming neighbor lists message $(u, N_u^{IN,1}, \ldots, N_u^{IN,max})$, which is sent at maximum power. When receiving the incoming neighbor lists messages from the neighbors, a node builds the set of outgoing neighbor lists. Once the outgoing neighbor lists are built, they are included in the outgoing neighbor lists message $(u, N_u^{OUT,1}, \ldots, N_u^{OUT,max})$, which is again sent at maximum power. After a node has received the incoming and outgoing lists of all its neighbors, it has all the information needed to compute the interference number of all the edges incident into it. A more formal description of the protocol is reported in Figure 5.

Protocol InterEst is proven to correctly compute the interference number of every link in the graph under the assumption that the following property holds:

Definition 4 (Max power symmetric property)**.** *Let $G = (N, E)$ be the maxpower graph. The max power symmetric property is satisfied whenever for any node $u \in N$, the fact that v is within u's transmitting range at maximum power implies that u is also within v's range at maximum power.*

Note that the max power symmetric property is weaker than the assumption that all the nodes have the same maximum power and the wireless medium is symmetric. For instance, it might be the case that u's maximum power is 50 mW, v's maximum power is 100 mW, and the wireless medium is not symmetric (e.g. because v cannot reach u at power 50 mW, while the reverse holds), yet the max power symmetric property is satisfied.

Theorem 4. *If the max power symmetric property is satisfied, then protocol* InterfEst *is correct, i.e. after its execution every node in the network knows the interference number of every edge incident into it.*

Proof. In order to compute the interference number of edge $e = (u, v)$, we have to compute the set of nodes within u's range when u transmits with the minimum power needed to reach node v, and the same set of nodes relative to v. The minimum power needed by v to reach u is power P_i, where i is such that $v \in N_u^{IN,i}$. Given the max power symmetric property, and the fact that nodes send the incoming neighbor lists at maximum power (step 3a. of the protocol), node v can correctly compute its sets of outgoing neighbors (step 3b. of the protocol). This information is sent to node u at step 5a. Again, the circumstance that node u correctly receives the outgoing neighbor lists of node v is implied by the max power symmetric property, and by the fact that node v sends the message containing its outgoing neighbor lists at maximum power.
So, at step 6 node u can compute the set of nodes within v's range at power level P_i as $N_v = \bigcup_{h=1}^{i} N_v^{OUT,h}$. In order to determine the minimum power level P_j needed by u to reach node v, node u inspects its set of outgoing neighbor lists, computed at step 3b. Note that, since v is within u's maximum range (otherwise, v would have been removed from the set of incoming neighbors at step 3b.), there exists j such that $v \in N_u^{OUT,j}$. It follows that all the nodes in $N_u = \bigcup_{h=1}^{j} N_u^{OUT,h}$ are within u's range at power P_j, and must be included in the interference count. So, the interference number is correctly computed at step 6. of the protocol as $|N_v \cup N_u|$.

To end the proof of the theorem, we need to show that, after InterfEst execution, u knows the interference number of all the edges incident into it. We recall that we are interested in estimating the interference of bi-directional links only. Let us then consider any bi-directional link $e = (u, v)$. Since the link is bi-directional, u

Protocol InterfEst:
(protocol for node u)

$N_u^{IN,i}$ denotes the set of incoming neighbors of node u at power level P_i;

$N_u^{OUT,i}$ denotes the set of outgoing neighbors of node u at power level P_i;

0. *Initialization*
    for i=1 to max do
        $N_u^{IN,i} = \emptyset$
        $N_u^{OUT,i} = \emptyset$
1a. *Send beacons*
    for i=1 to max
        send beacon (u, P_i) at power level P_i
1b. *Receive beacon*
    upon receiving beacon message (v, P_j)
        if first beacon message received from v
            $N_u^{IN,j} = N_u^{IN,j} \cup \{v\}$
2. *Wait for the stabilization period*
3a. *Send incoming neighbor lists*
    send message $(u, N_u^{IN,1}, \ldots, N_u^{IN,max})$
    at power level P_{max}
3b. *Receive incoming neighbor lists*
    upon receiving message $(v, N_v^{IN,1}, \ldots, N_v^{IN,max})$
        store the incoming neighbor lists of node v
        find i such that $u \in N_v^{IN,i}$
        if no such i exists then
            delete v from the incoming neighbor lists (v is out of u's maximum range)
        otherwise
            $N_u^{OUT,i} = N_u^{OUT,i} \cup \{v\}$
4. *Wait for the stabilization period*
5a. *Send outgoing neighbor lists*
    send message $(u, N_u^{OUT,1}, \ldots, N_u^{OUT,max})$ at power level P_{max}
5b. *Receive outgoing neighbor lists*
    upon receiving message $(v, N_v^{OUT,1}, \ldots, N_v^{OUT,max})$
        store the outgoing neighbor lists of node v
6. *Compute interference numbers*
    for $i = 1$ to max
        for each $v \in N_u^{IN,i}$ do
            $N_v = \bigcup_{h=1}^{i} N_v^{OUT,h}$
            find j such that $v \in N_u^{OUT,j}$
            $N_u = \bigcup_{h=1}^{j} N_u^{OUT,h}$
            $IN(e = (u, v)) = |N_v \cup N_u|$

Figure 5: Protocol for estimating the interference number in heterogeneous networks.

is within v's maximum range, and vice versa. It follows that there exists i such that $v \in N_u^{IN,i}$. Furthermore, since v is within u's maximum range, there exists j such that $u \in N_v^{IN,j}$. In turn, this implies that $v \in N_u^{OUT,j}$. It follows that the interference number of edge e can be correctly computed by both node u and node v. □

Note that the max power symmetric property is essential for the correct computation of the interference number. In fact, consider the case where u is reachable from v at v's maximum power, but v cannot be reached by node u at u's maximum power. When v calculates the interference number of the edges on which it communicates at maximum power, it should include u in the count, since v's transmission at maximum power would interfere with node u. However, v will never receive any protocol message from u (because it is out of u's maximum transmitting range), so it has no way of knowing that node u exists and it cannot possibly include u in the interference count.

In case the max power symmetric property does not hold, it can be easily seen that the InterEst protocol computes a lower bound to the interference number of the links.

Proposition 1. *If the max power symmetric property does not hold, then protocol* InterfEst *computes a lower bound to the interference number of all the bi-directional links in the maxpower graph.*

Note that even in case the max power symmetric property does not hold, InterEst is likely to compute the exact value of the interference number on most network links. In fact, the possible inaccuracy in interference number computation applies only to those links (u, v) such that: (i) there exists a third node w in v's range at power P_i (which is the minimum power needed to sustain link (u, v)), (ii) v is out of w's transmitting range at maximum power. A consequence of this observation is that the inaccuracy is likely to occur on links which have a relatively high interference number, which will probably be discarded by, say, an interference-aware routing protocol even in case of an incorrect (lower) computation of the interference number. Summing up, we can conclude that InterEst can be used to accurately estimate the expected interference on a link also in networks that do not satisfy the max power symmetric property.

Theorem 5. *Protocol* InterfEst *has $O(n(max + 2))$ message complexity, which is at most a factor $O(\Delta max)$ away from optimal, where Δ is the maximum node degree in the maxpower graph.*

Proof. The proof is along the same guidelines as the proof of Theorems 2 and 3. □

5.3 Implementation issues

The correctness of the protocols introduced in this section is based on the assumption that all messages (beacon and neighbor list messages) sent by neighbors are correctly received by any node within a certain finite time period, which we called stabilization period. An important issue to address when implementing our protocols is then how to set the stabilization period. However, prior to this, we want to remark that a single execution of our protocols should be better intended as a stage of an "interference estimation" protocol, which is executed periodically in order to account for changes in the network topology and/or in the wireless links conditions. Hence, HInterfEst and InterfEst must be considered as protocols for *estimating* (and not exactly computing once and for all) the interference number on the links. In this respect, it is possible to tolerate the loss or late arrival (after the stabilization period is expired) of some messages, which can be sent again and received (in case of lost messages) or accounted for (in case of late arrival) during the next execution of the protocol.

Having established the above, the stabilization period can be estimated based on a knowledge of the maximum number of one-hop neigbhors (at maximum power) of the network nodes. In turn, the maximum number Δ of neighbors can be upper bounded by knowing the node maximum transmit range and the average node density, which are typically known, at least with a certain degree of approximation, to the network designer. Once an upper bound on Δ is known, the stabilization period can be determined by estimating the time required by the MAC layer protocol at hand to serialize Δ transmissions across the wireless channel.

6 TC for multi-hop interference

As discussed above, the notion of interference used in the current TC literature does not account for interference in multi-hop communications. A consequence of this fact is that MST-like topologies (as they are computed by the LIFE algorithm introduced in [2], and by the various algorithms introduced in [13]) are claimed to be optimal for reducing interference. *The following theorem shows that this claim is false if the multi-hop nature of communications in ad hoc networks is accounted for.*

Theorem 6. *Assume the nodes have a circular radio coverage area (in this case, interference number and coverage are equivalent notions). Let $G = (N, E)$ be the maxpower graph, and assume G is connected. Let $MST = (N, E_{MST})$ be a MST built on G using the interference number as the edge weight. The PIC spanning factor of the MST is $\Omega(n)$, where $n = |N|$.*

Proof. Consider the node placement reported in Figure 6. Assume the nodes' maximum transmitting range is d. Nodes u and v are at distance d from each other. The remaining $n-2$ nodes form a chain, where consecutive nodes in the chain are at distance $d' < d$ from each other. Furthermore, we have the property that any two non consecutive nodes in the chain are at distance greater than d. The endpoints of the chain are nodes w_1 and w_{n-2}, where w_1 is at distance d' from u and at distance $> d$ from v, and w_{n-2} is at distance d' from v and at distance $> d$ from u. Furthermore, there is a third node, w_2, which is at distance d'' from u, with $d' < d'' < d$, and at distance $> d$ from v. All the other nodes in the chain are out of u's and v's maximum transmitting range. The resulting maxpower graph is reported in Figure 6; in the figure, edges are labeled both with their length and with the interference number.

With this node configuration, G is a connected graph composed of $n+1$ edges: edges (u, v) and (u, w_2) have interference number equal to 5, edges (u, w_1) and (v, w_{n-2}) have interference number equal to 3, and the remaining edges have interference number equal to 4. When computing the MST, all the edges of weight < 5 are considered before edges (u, v) and (u, w_2) are taken into account. Since the subgraph of G obtained by considering all the edges with weight 3 and 4 is connected, it follows that links (u, v) and (u, w_2) are not included in the MST. The MST resulting from this node configuration is represented by bold edges in Figure 6. The minimum interference path connecting u and v in the MST has cost $2 \cdot 3 + 4 \cdot (n-3)$; on the other hand, the minimum interference

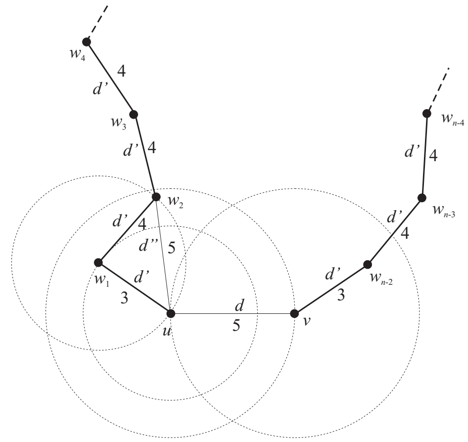

Figure 6: Example showing that the interference-based MST has $\Omega(n)$ PIC spanning factor.

path between u and v in G is edge (u, v), whose cost equals 5. Thus, we can conclude that the PIC spanning factor of the interference-based MST is $\Omega(n)$, and the theorem is proven. □

The authors of [2] introduced other low-interference topologies which, besides preserving connectivity, are good spanners. However, they consider Euclidean spanners, which in general are not good at reducing multi-hop interference.

7 The ATASP topology

In the previous section we have proved that MST-like topologies are not appropriate for reducing multi-hop interference. What is then a good topology for this purpose? The following analysis answers this question.

Definition 5 (ATASP topology)**.** *Let $G = (N, E)$ be the maxpower graph. The (interference) ATASP subgraph of G is the graph with node set N and edge set E_{ATA}, where edge $(u, v) \in E_{ATA}$ if and only if there exists a source/destination pair w, z in N such that edge (u, v) belongs to a minimum interference path connecting w and z in G.*

The intuition behind the notion of ATASP (All-To-All-Shortest-Path) graph is the following: in principle, an edge e can be declared "inefficient", and thus removed from the final network topology G', only if it is not part of any interference-optimal path in the graph. Otherwise, removing e from the network topology might increase the PIC of some optimal source/destination path, possibly leading to an increase of the PIC spanning factor of G'.

Note that the increase in the PIC spanning factor does not necessarily occur: in fact, it might be the case that there exist multiple minimum interference paths connecting two nodes, and removing an edge along one of these paths does not increase the PIC spanning factor. However, with the definition of ATASP graph introduced above, we are ensured that in every possible node placement ATASP has optimal PIC spanning factor. This is stated in the following theorem.

Theorem 7. *Let G be the maxpower graph, and let ATASP be the graph constructed as in the definition above. ATASP has optimal PIC spanning factor, i.e.,* $\rho(ATASP) = 1$.

Proof. The proof follows immediately by the definition of ATASP graph. □

The fact that ATASP has optimal PIC spanning factor implies that it preserves worst-case connectivity:

Theorem 8. *Let G be the maxpower graph, and let ATASP be the graph constructed as in the definition above. Then ATASP is connected iff G is connected.*

Although the ATASP topology has the nice features of being an optimal PIC spanner and of preserving network connectivity, the question of whether ATASP is actually a sparse subgraph of G remains open. The following theorem gives a negative answer to this question.

Theorem 9. *There exist a node configuration and maximum transmit power setting such that the maxpower graph G is composed of $\Theta(n^2)$ edges, and its ATASP subgraph is composed of $\Theta(n^2)$ edges as well.*

Proof. Consider a placement of n equally spaced nodes on a circle, numbered consecutively 0 through $n-1$. Suppose also that the maximum transmitting range of a node is not less than the diameter of the circle, so that the maxpower graph $G = (N, E)$ is the complete graph. We prove that, for any $u, v \in N$, the arc (u, v) must be in the ATASP topology, i.e., $E_{ATA} = \Theta(n^2)$.

By a trivial symmetry argument, the cost of the optimum interference path between any two nodes only depends on their *distance* k, measured as the minimum number of nodes between them (moving either clockwise or counterclockwise). Thus, we may assume, w.l.o.g, that $u = 0$ and $v = k$, with $0 < k \leq \frac{n-1}{2}$. A link between nodes $i < j$ is a *chord* of length c if $j - i = c$. It is easy to see that $IN((i,j)) = \min\{n, 3(j-i)+1\}$. In fact, a transmission along the chord (i, j) will interfere with the c nodes "preceding" the sender i, the c nodes "following" the receiver j, and the $c-1$ nodes in-between; counting also sender and receiver and summing up gives interference $3c+1$. See Figure 7.

Now, in looking for a minimum interference path between 0 and k, we can limit our search to monotonic paths, i.e., paths such that, for any intermediate transmission $s \rightarrow r$ (if any), $r > s$ holds true. This fact can be easily proven by induction on the value of k. But then, any optimum interference path p between 0 and k must satisfy the equation

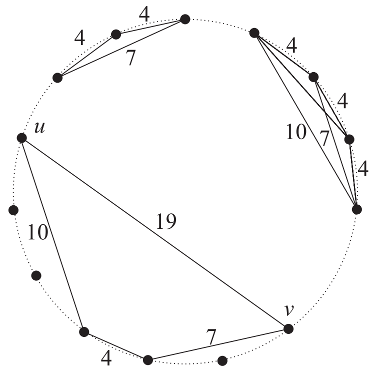

Figure 7: Placement of nodes for Theorem 9. In this example, $k = v - u = 6$.

$$\begin{aligned} PIC(p) &= \min_{\substack{0\le h<k \\ 0<i_1<\ldots<i_h<k}} PIC(0, i_1, \ldots, i_h, k) \\ &= \min_{\substack{0\le h<k \\ 0<i_1<\ldots<i_h<k}} \sum_{j=0}^{h} IN((i_j, i_{j+1})) \\ &= \min_{\substack{0\le h<k \\ 0<i_1<\ldots<i_h<k}} \sum_{j=0}^{h} (3(i_{j+1} - i_j) + 1) \end{aligned}$$

where we have set $i_0 = 0$ and $i_{h+1} = k$. The terms in the last summation telescope, giving $PIC(p) = 3k + h + 1$ which is minimized for $h = 0$, i.e., when p coincides with the link $(0, k)$. Since k is arbitrary, this means that *any* chord of length k must indeed be present in E_{ATA}. □

Indeed, the very same node placement adopted in the proof of Theorem 9 can be used to prove the following stronger negative result about multi-hop interference-based TC:

Corollary 1. *There exist a node placement and maximum transmit power setting such that no link can be removed from the maxpower communication graph without increasing multi-hop interference.*

In words, Corollary 1 states that *there exist situations in which performing multi-hop interference-based TC is useless, since all the links in the maxpower graph turn out to be interference-efficient.*

Is then performing multi-hop interference-based TC pointless, as it is the case of energy-based TC? To answer this question, we first observe that Theorem 9 and Corollary 1 refer to a worst-case scenario, which is quite unlikely to occur in practical situations. To gain insights on the ATASP sparseness in average-case situations, we have estimated the average node degree in ATASP through extensive simulations on randomly deployed networks. To generate the maxpower graph G, a number n of nodes is distributed uniformly at random in the unit square, and the maxpower graph is computed according to a certain radio channel model. Similarly to the simulations for energy, we have considered values of n ranging from 10 to 500 nodes, and two radio channel models: the quite idealistic free space propagation model, and the log-normal shadowing model.

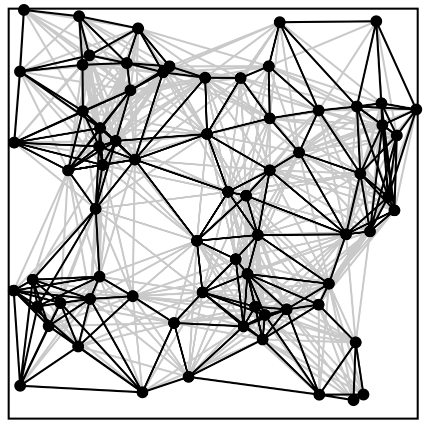

Figure 8: Sample of ATASP graph. The radio channel model is free space.

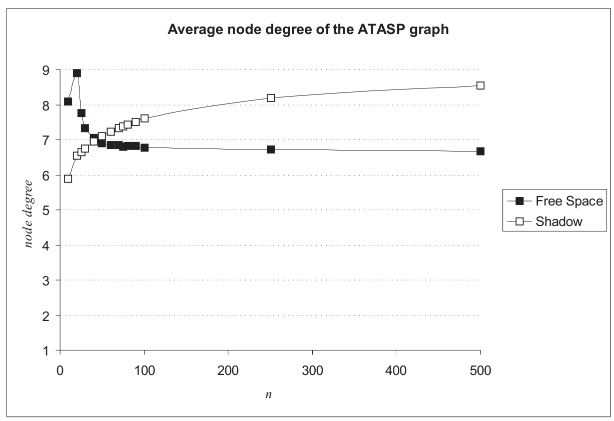

Figure 9: Average node degree of the ATASP graph for increasing network size with free space propagation, and with log-normal shadowing ($\sigma = 6$). Node distribution is uniform.

Once the maxpower graph has been generated, we assign weights to the links according to the interference number, and we compute the optimal all-to-all shortest paths. Every edge which is part of at least one such paths is marked as belonging to ATASP. At the end of this process, the ATASP topology is computed, and the average node degree recorded. A sample of ATASP topology is reported in Figure 8.

The results of our simulations are reported in Figure 9. As seen from the figure, the average degree with log-normal shadowing is slightly higher than the degree with free space propagation. However, in both cases the average degree remains confined below 8.5, even for large networks.

To evaluate the effect of node concentration on the average ATASP node degree, we have repeated the simulations using the two-dimensional Normal distribution to deploy nodes. The simulation results, which are reported in Figure 10, show that the effect of node concentration on the ATASP node degree is marginal.

Overall, simulation results show that, while ATASP is a dense graph (actually, it can coincide with the

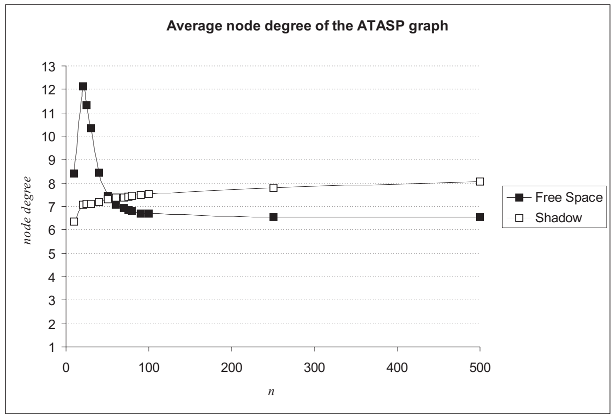

Figure 10: Average node degree of the ATASP graph for increasing network size with free space propagation, and with log-normal shadowing ($\sigma = 6$). Node distribution is Normal.

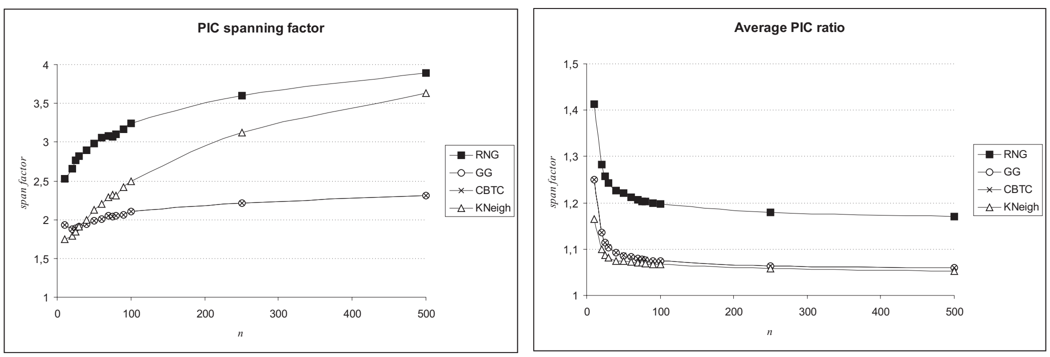

Figure 11: PIC spanning factor (left) of different localized topologies with free space propagation. The graphic on the right show the average PIC ratio. Node distribution is uniform.

maxpower communication graph) in the worst case, it is a sparse subgraph of the maxpower graph on the average, indicating that, if we exclude pathological node placements, multi-hop interference-based TC is actually possible.

8 Localized low-interference topologies

In the previous section we have identified ATASP as the interference-optimal topology, under the assumption that multi-hop interference is considered. Unfortunately, building the ATASP graph requires global knowledge, thus impairing one of the desired features of topology control protocols, i.e., locality.

While we leave the problem of designing a localized TC protocol for building a provably multi-hop interference optimal topology open, in this section we investigate through simulation how do existing localized topologies, which have been proposed in the literature with the purpose of reducing energy consumption (based on a quite

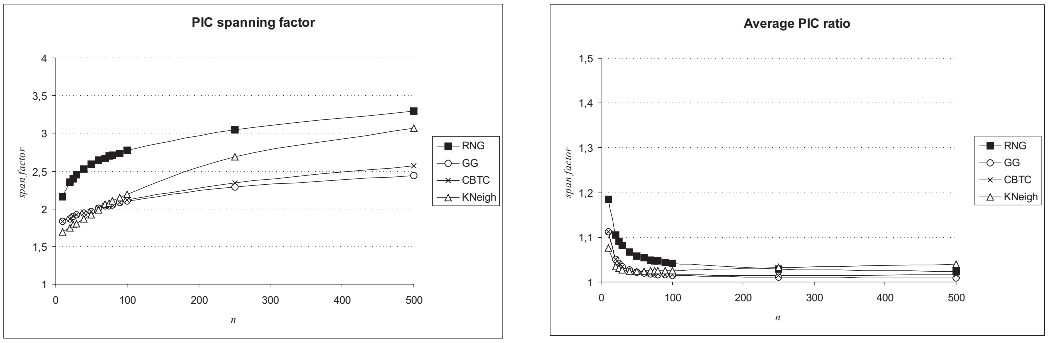

Figure 12: PIC spanning factor (left) of different localized topologies with log-normal shadowing. The graphic on the right show the average PIC ratio. Node distribution is uniform.

unrealistic energy model – see Section 2), perform with respect to multi-hop interference.

The simulation setting is the same as in experiments reported in the previous sections: n nodes randomly distributed in the unit square (uniform or normal distribution), values of n ranging from 10 to 500, and free space or log-normal shadowing radio channel model.

We have then considered four different topologies built on the maxpower graph: the Relative Neighbor Graph, the Gabriel Graph, the CBTC graph [17], and the KNeigh graph [1]. For each of these graphs, we have computed the PIC spanning factor with respect to the original maxpower graph.

Note that, in case of log-normal shadowing propagation, we have partially modified the definitions of RNG, GG, CBTC and KNeigh graph: instead of considering the actual node distances to compute the graphs, we have considered the "virtual" distance obtained by accounting for the shadowing effect (see Section 2). For instance, in the KNeigh protocol, instead of connecting each node to its k closest neighbors, we have connected each node to the k neighbors which can be reached with the less power, independently of the actual distance to these nodes. From a worst-case perspective, this modification might cause these graphs to loose their connectivity property (we recall that RNG, GG and CBTC preserve worst-case connectivity, while KNeigh preserves connectivity with high probability). However, our simulations show that this unfortunate situation is very likely not to occur.

The results of our simulations are shown in Figure 11 for the case of free space propagation, and in Figure 12 for the case of log-normal shadowing with uniform node distribution. Besides computing the PIC spanning factor, we have also computed the *average* ratio of the cost of the interference optimal path in the topology at hand to the cost of the optimal path in G (we recall that the PIC spanning factor is the maximum of these ratios). This value, which we call the average PIC ratio, gives an idea of the average interference penalty caused by using a certain subgraph of G to route messages.

As seen from the figures, the PIC spanning factor of all the topologies considered remains confined below 4 in case of free space propagation, while it remains below 3.5 with log-normal shadowing. The topology that shows the best performance is the GG, with a PIC spanning factor below 2.5 with both free space propagation

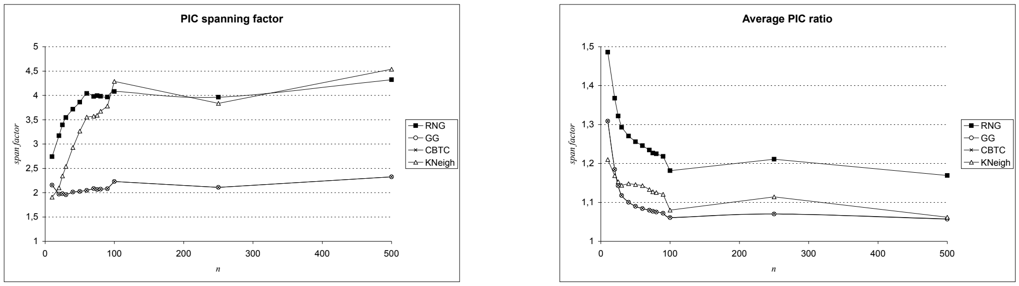

Figure 13: PIC spanning factor (left) of different localized topologies with free space propagation. The graphic on the right show the average PIC ratio. Node distribution is Normal.

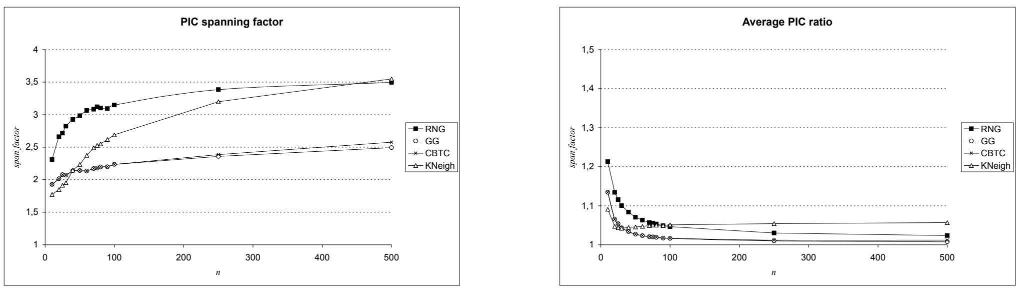

Figure 14: PIC spanning factor (left) of different localized topologies with log-normal shadowing. The graphic on the right show the average PIC ratio. Node distribution is Normal.

and log-normal shadowing. When considering the average PIC ratio, the situation is even better: the average interference penalty of the topologies considered is below 1.2 for moderate to large size networks with free space propagation, and it is below 1.1 with log-normal shadowing. The GG is the best performing topology also with respect to this metric.

The simulation results obtained with Normally distributed points, which are reported in Figures 13 and 14, show that the effect of node concentration on the PIC spanning factor and on the average PIC ratio of the different topologies is scarcely significant.

9 The triangular inequality and interference

In the previous section, we have shown that localized topologies that have been introduced in the literature with the purpose of reducing energy consumption (under an unrealistic energy model – see Section 2) turn out to perform well with respect to multi-hop interference. Among these topologies, the GG is the one that displays best performance.

In this section, we argue that this fact happens by no chance, but it is a consequence of the triangular inequality argument, which, although not valid as far as energy is concerned, turns out to hold (under certain assumptions) for multi-hop interference.

Assume that the radio coverage area is a perfect circle, and that nodes are randomly, densely distributed. In particular, we model the distribution of nodes in (2-dimensional) space according to the Poisson process with given density parameter λ. In this scenario, let $N(S)$ denote the number of nodes in the surface S, and let $\mu(S)$ denote the measure (i.e., area) of S; then

$$\mathrm{Prob}(N(S) = k) = e^{-\lambda\mu(S)} \frac{(\lambda\mu(S))^k}{k!}. \tag{2}$$

Also, if S_1 and S_2 are disjoint surfaces, the variables $N(S_1)$ and $N(S_2)$ are independent.

Theorem 10. *Let u and v be two adjacent nodes in the communication graph and let $IN((u,v))$ denote the interference number of the edge (u,v). Let the nodes of the wireless network be distributed according to the Poisson process in space with density λ. Then*

$$\mathrm{P}(IN((u,v)) = k) = \mathrm{e}^{-\lambda\mu(S_{uv})} \frac{(\lambda\mu(S_{uv}))^k}{k!},$$

where S_{uv} is the surface depicted in Figure 2.

Proof. (Sketch) The result is quite intuitive. Given a distribution of nodes, we pick the edge (u,v) whose interference we want to compute. The probability that the region S_{uv} contains k nodes "should be" the same as the probability that $S_{uv} \backslash (\{u\} \cup \{v\})$ contains k nodes, since the set $\{u\} \cup \{v\}$ has measure 0. □

The actual value of $\mu(S_{uv})$ can be easily computed as twice the area of the circle of radius $r_{uv} = \mathrm{dist}(u,v)$ minus the area C_{uv} of the intersection of two such circles whose centers are at distance r_{uv} (see Figure 2). Because of the symmetry, the latter can be computed as follows:

$$C_{uv} = 4 \cdot \int_0^{\frac{\sqrt{3}}{2} r_{uv}} \left(\sqrt{r_{uv}^2 - x^2} - \frac{1}{2} r_{uv} \right) \mathrm{d}x = \\ = \left(\frac{2}{3}\pi - \frac{\sqrt{3}}{2} \right) r_{uv}^2 \ .$$

Twice the area of the circle minus the above value gives then

$$\mu(S_{uv}) = 2\pi r_{uv}^2 - C_{uv} = \left(\frac{4}{3}\pi + \frac{\sqrt{3}}{2} \right) r_{uv}^2 = \gamma r_{uv}^2 \ ,$$

where $\gamma \approx 5.0548$.

Given two adjacent nodes u and v, it is not easy to compute the probability of the following event: the interference over (u,v) is smaller (larger) than the sum of the interferences over the edges (u,w) and (w,v), where w is a third node adjacent to both u and v. In fact, the events "number of nodes in S_{xy}" (where $x, y \in \{u, v, w\}$ and $x \neq y$) are highly dependent.

On the side of expectations, though, the computation is straightforward. In fact, we have

$$\begin{aligned} \mathrm{E}[N_{uw} + N_{wv}] &= \mathrm{E}[N_{uw}] + \mathrm{E}[N_{wv}] = \\ \lambda\mu(S_{uw}) + \lambda\mu(S_{wv}) &= \lambda\gamma(r_{uw}^2 + r_{wv}^2) \end{aligned}$$

Analogously, $\mathrm{E}[N_{uv}] = \lambda\gamma r_{uv}^2$, and thus $\mathrm{E}[N_{uw} + N_{wv}] \leq \mathrm{E}[N_{uv}]$ if and only if $r_{uw}^2 + r_{wv}^2 \leq r_{uv}^2$. This amounts to saying that w must not lay within the circle having the edge (u, v) as the diameter. This corresponds exactly to the definition of Gabriel Graph.

10 Conclusions

In this paper we have demonstrated the importance of accurately choosing the energy and interference model when studying the topology control problem in wireless ad hoc networks. While we do not promote ours as the best possible energy and multi-hop interference models for ad hoc networks, we believe that they capture the features of this type of networks better than the models used in the literature so far. As a consequence, we believe the conclusions about TC presented in this paper are closer to reality than the ones presented in previous work. We are currently working on setting up an experimental testbed for some of the TC techniques considered in this paper, in order to experimentally validate our findings. This testbed could also be used to demonstrate the capability of topology control to increase network throughput in a realistic setting.

While this and other recent papers represent progress on the topic of minimizing interference in ad hoc networks, much work remains to be done on this topic. There are also many interesting open questions surrounding the interplay between interference, energy, delay, and throughput.

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