# Topology Control with Better Radio Models: Implications for Energy and Multi-Hop Interference

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## ABSTRACT

Topology Control (TC) is a well-studied technique used in wireless ad hoc networks to find energy-efficient and/or low-interference subgraphs of the maxpower communication graph. However, existing work has the following limitations: (1) the energy model adopted is quite unrealistic - only transmit power is often considered and homogeneous decay of the radio signal with distance is assumed; (2) the interference measure does not account for multi-hop communications. In this paper, we show the dramatic effect of the underlying energy and interference model on TC. In particular, we demonstrate that by using more realistic energy models and considering the effects of multi-hop interference, radically different conclusions about TC can be drawn; namely that (1) energy efficient TC is essentially meaningless, since every link turns out to be "efficient", and that (2) topologies identified as "interference-optimal" in the current literature can be extremely bad from the viewpoint of multi-hop interference. Given these observations, we propose a new measure of link interference, extend it to deal with multi-hop interference, and design a corresponding optimal communication subgraph, called ATASP. We prove that, in the worst case, ATASP coincides with the maxpower communication graph, showing that in some unfortunate situations also performing multi-hop interference-based TC is pointless. However, the simulation results with random node deployments presented in this paper show that, on the average, ATASP is a sparse subgraph of the maxpower communication graph, and multi-hop interference-based TC is indeed possible. Since computing ATASP requires global knowledge, we experiment through simulation with known localized algorithms for energy-efficient TC and show that they perform well (on the average) with respect to multi-hop interference.

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Performance, Experimentation, Algorithms

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Ad hoc wireless networks, topology control, radio models, interference

## 1. INTRODUCTION

Topology Control (TC) attempts to find efficient but sparse subgraphs of the maxpower communication graph in a wireless ad hoc network [1, 2, 10, 12, 14, 16]. The goal of TC is to eliminate inefficient links that ought not be used for communication. In addition to TC's inherent benefits, use of a sparse topology reduces routing overhead, which can be quite high in ad hoc networks due to expensive flooding of route discovery messages [9].

The efficiency metrics used to date in the TC literature are: (1) energy [1, 10, 14, 16] and (2) interference [2, 12]. The need to reduce energy is fundamental in energy-constrained environments, while reducing interference has the potential to increase network capacity [6, 7, 8].

However, existing work has made some significant simplifying assumptions. First, when considering energy, only transmit power is typically considered and it is assumed that power decays as $\frac{1}{d^\alpha}$, where $d$ is the distance between sender and receiver and $\alpha$ is the path loss exponent. This is known to be a poor model for energy consumption of the entire network interface (as we demonstrate in Section 2). Also when the receiver power is accounted for (as in [14]), the assumption of homogeneous power decay with distance is used, implying that the transmit power varies from nearly 0 (when the receiver is very close to the sender) to high values (when the receiver is far away). Actually, as we discuss in Section 2, in real wireless transceivers the ratio between the minimum and the maximum possible transmit power is limited, and it is often well within a factor 2. As we will discuss therein, accounting for the actual ratio between the minimum and the maximum possible transmit power leads to draw radically different conclusions about which links are energy-efficient.

Simplifying assumptions have been made also when considering interference, namely that (1) the transmission regions are perfectly circular and (2) interference in multi-hop communications is not accounted for.

In this paper, we study the TC problem using more realistic energy and interference models, and we show that if such models are used, radically different conclusions about TC are drawn.

Concerning energy, we show that, at least with current transceiver technology, no energy-efficient TC is possible: every link in the communication graph is energy-efficient. This statement is first theoretically proved for the case of three nodes (reversing the well-known triangular inequality argument), and then validated through simulation for larger network sizes.

Concerning interference, we show that: $(i)$ MST-based topologies (proposed as optimal solutions in current literature [2, 12]) are actually $\Omega(n)$ away from the optimal solution ($n$ is the number of network nodes) if multi-hop interference is accounted for; and $(ii)$ there exist node placements and transmit power settings such that removing any link from the maxpower communication graph results in increasing multi-hop interference.

In light of $(ii)$, one might conclude that no multi-hop interference TC is possible as well. However, $(ii)$ holds in a worst-case scenario. Is some type of TC possible for non-pathological node placements? To answer this question, we propose a new network topology, called ATASP, which is shown to be optimal from the point of view of multi-hop interference (i.e., it maintains all the interference-efficient links), and we investigate the properties of this topology through simulation with random node deployments. The results of our simulations show that, if we exclude pathological node placements, multi-hop interference-based TC is actually possible, since most of the links in the communication graph can be removed without increasing multi-hop interference.

Unfortunately, computing ATASP requires global knowledge. While we leave the problem of designing a localized TC protocol for building a provably multi-hop interference optimal topology open, we show through simulation that some of the localized protocols proposed for energy-efficient TC actually perform quite well (on the average) with respect to multi-hop interference.

We believe the main contribution of this paper is to make it clear the dramatic impact of the underlying energy and interference model used on the conclusions that can be drawn about the network topology. While it was well known in the community that using "simple" models could lead to "inaccurate" conclusions about the optimal network topology, no research has thoroughly investigated the relations between radio models and the resulting optimal network topology. The results presented in this paper clearly demonstrate the importance of choosing a realistic (although necessarily simplified) radio model when studying fundamental properties of wireless ad hoc networks.

## 2. TC FOR ENERGY

In this section, we use the following notation for the power consumption parameters of a network interface: $t_{\max}$ = transmit power at highest setting; $t_{\min}$ = transmit power at lowest setting; $r$ = receive power.

Consider Figure 1, which shows a typical situation involving three nodes having a triangle relationship.

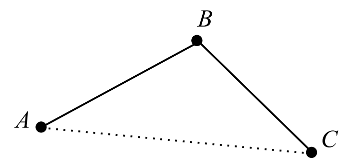


**Figure 1: Example of a Triangle Relationship**

In this situation, we are concerned with sending data from node A to node C and we would like to decide whether it is more energy-efficient to use the direct link connecting the two nodes or to use the multi-hop path with two links $(A, B)$ and $(B, C)$. We are primarily interested in the question of whether it is ever more energy-efficient (with realistic network interface parameters) to use the multi-hop path. Accordingly, we consider the best-case scenario for the multi-hop path, i.e., the situation where links $(A, B)$ and $(B, C)$ use $t_{\min}$ and link $(A, C)$ uses $t_{\max}$.

Considering both transmit and receive powers, the total power for the single-hop path is $t_{\max} + r$ and the total power for the multi-hop path is $2t_{\min} + 2r$. Thus, the multi-hop path is preferable if and only if:

$$t_{\min} < \frac{t_{\max} - r}{2} \quad (1)$$

Much of the existing work assumes $t_{\max} \gg t_{\min}, r$. In this situation, Inequality 1 would hold. However, this assumption accounts only for the power consumed by the power amplifier but not the *total* power consumed by the interface.

Data from all network interfaces that we have seen show that $r$, $t_{\min}$, and $t_{\max}$ are all within a factor of two. Triangle Inequality (1) clearly does not hold for values in this range. For example, in the Cisco Aironet 4800 card, $r = 0.958t_{\min}$ and $t_{\max} = 1.358t_{\min}$ [4]. For Inequality (1), these values make the left hand side 2.5 times greater than the right hand side. In the sensor domain, the Medusa II sensor nodes have $r = 1.107t_{\min}$ and $t_{\max} = 1.265t_{\min}$ [13]. Here, the situation is even less favorable for the multi-hop path in that the left hand side becomes almost 13 times as large as the right hand side!

These simple analyses have a serious implication on topology control for energy reduction. *Because the most energy-efficient path between two nodes that are the endpoints of a wireless link is the link itself, no link is unnecessary if minimum-energy paths are to be used at all times and thus, no topology control is possible*[1].

[1]Note that this statements holds with current transceiver technology, and it might no longer hold when the technology will allow to have $t_{min}$ order of magnitudes lower than $t_{max}$.

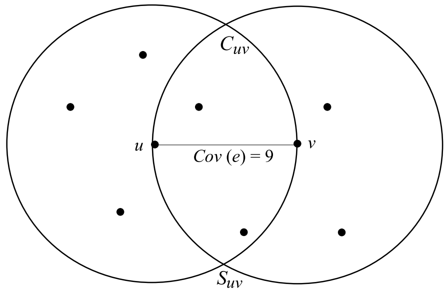


**Figure 2: Coverage measure of the edge $e = (u, v)$.**

These analyses leave open the theoretical possibility that a path connecting two nodes of length $k+1$ is more energy-efficient than a path between the same nodes of length $k$, for large enough $k$. However, in the following simulation results (with the Cisco Aironet 4800 power values), this situation never occurred. The minimum-energy path between two nodes *always* corresponded to a minimum-hop path. This implies that, from a practical standpoint, *energy-aware routing corresponds to selecting the minimum-energy path from among all minimum-hop paths.*

The simulations have been performed considering $n$ nodes ($n$ ranges from 10 to 500) deployed uniformly at random in the unit square. Two radio channel models have been considered: free space propagation (circular radio coverage, with path loss exponent $\alpha = 2$), and log-normal shadowing. In the log-normal shadowing model, the transmitted signal attenuation at a certain distance is determined by the sum of a deterministic and a random component. This way, the log-normal shadowing model accounts for the situations in which the radio coverage area is irregular. The deterministic component gives the average value of the received signal, which is determined by the distance between sender and receiver and by the path loss exponent (set to 2 in our experiments). The random component has log-normal distribution (normal distribution when measured in dBs) with standard deviation $\sigma$ ($\sigma = 6$ in our experiments).

Note that the log-normal shadowing model defines a *virtual distance* between two nodes, which results from the combination of the deterministic and the random components of the signal attenuation. We can say that two nodes in the log-normal shadowing model are neighbors if and only if their *virtual distance* is below the maximum transmitting range.

The nodes maximum transmitting range in our simulations was set to the value of the critical transmitting range for connectivity, augmented by 50% (see [15]).

The energy cost of link $(u, v)$ is computed according to the following formula:

$$EC(u,v) = E_r + E_{txMin} + (E_{txMax} - E_{txMin}) \left( \frac{dist(u,v)}{Tr} \right)^{\alpha} ,$$

where $dist(u, v)$ is the distance (in case of log-normal shadowing, the virtual distance) between nodes $u$ and $v$, $Tr$ is the maximum transmitting range, $E_r$ is the energy consumed in receiving a packet, and $E_{txMin}$ and $E_{txMax}$ are the energy consumed at minimum and maximum transmit power, respectively. The values of $E_r$, $E_{txMin}$ and $E_{txMax}$ are taken from [4].

To evaluate the effect of node concentration on the minimum-energy paths, we have repeated the simulations using the two-dimensional Normal distribution to deploy nodes. Indeed, we have considered only the nodes which are deployed in the unit square: that is, to generate a network with $n$ nodes, we distribute nodes according to the two-dimensional Normal distribution, discarding the node if it falls outside the unit square. In general, we thus need the generation of $n_1 > n$ nodes to build a network with $n$ nodes.

As anticipated above, in all the simulated scenarios, the minimum-energy path between two nodes *always* corresponded to a minimum-hop path.

## 3. TC FOR INTERFERENCE

The first paper that explicitly addresses the problem of interference-based topology control is [2]. In this work, Burkhart et al. define a metric that estimates the possible interference by a communication along the link. They call this measure *coverage*, which is formally defined as follows:

DEFINITION 1. *Let $e = (u, v)$ be any edge of the communication graph $G = (N, E)$, indicating that nodes $u, v \in N$ are within each other's maximum transmitting range. The coverage of edge $e$ is defined as*

$$Cov(e) = |\{w \in N : w \text{ is inside } D(u, \delta(u,v))\} \cup \{w \in N : w \text{ is inside } D(v, \delta(u,v))\}| ,$$

*where $D(x, y)$ denotes the disk of radius $y$ centered at node $x$, and $\delta(x, y)$ is the distance between $x$ and $y$.*

The example reported in Figure 2 clarifies the definition of edge coverage.

Based on the notion of link coverage, Burkhart et al. define the *interference* of a certain communication graph $G = (N, E)$ as the maximum coverage over all possible links. Formally, $I(G) = \max_{e \in E} Cov(e)$.

Given this notion of graph interference, the authors of [2] identify a set of sparse, connected topologies that minimize interference.

Other interference measures have been proposed in [12]. In particular, Moaveni-Nejad and Li define the interference of a graph as the *average* of the link coverage. Formally,

$$AI(G) = \frac{\sum_{e \in E} Cov(e)}{|E|} .$$

We observe that the notions of graph interference introduced in the literature so far suffer two major problems: ($i$) they are based on the notion of link coverage, which is purely geometric; and ($ii$) they do not account for interference in multi-hop communications.

Problem ($i$) implies that the link coverage is an accurate measure of the expected interference only under particular circumstances, i.e., when the radio coverage area can be modeled as a perfect circle. Unfortunately, this is not the case in most practical situations, due to shadowing and fading effects.

Problem ($ii$) can be even more serious, since most communications in ad hoc networks are expected to occur along multi-hop paths. As we shall see, not accounting for multi-hop interference might lead to radically different conclusions about which is the "interference-optimal" topology.

In the next sections, we propose solutions to address these two problems.

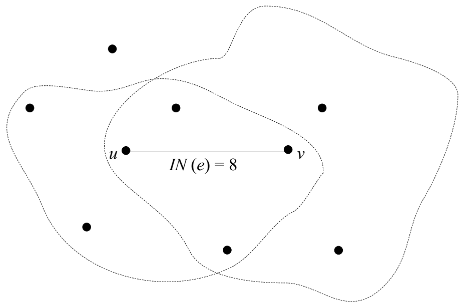


**Figure 3: Interference number of the edge $e = (u, v)$.**

# 4. THE INTERFERENCE NUMBER

In this section, we introduce a new link metric for estimating interference, which is a generalization of coverage, and we propose a metric to measure interference in multi-hop communications.

As observed in the previous section, the definition of coverage is purely geometric, and it relies on the assumption of perfect circular coverage of the radio signal. That is, this definition relies on a specific radio channel model, which does not account for shadowing and fading effects. Other notions of interference have been recently proposed in [11] and in [12], but they are similar to coverage in that they also are purely geometric definitions and rely on a specific radio channel model.

To circumvent this problem, we generalize the definition of coverage introduced in [2], obtaining a new measure of the interference associated with a link. The most notable aspect of this definition is that *it does not rely on the strong and often unrealistic assumption that the radio coverage area is a perfect circle*. Thus, it can be used in combination with more general radio channel models, which account for shadowing/fading effects.

DEFINITION 2 (INTERFERENCE NUMBER). *Let $e = (u, v)$ be any edge of the communication graph $G = (N, E)$, indicating that nodes $u, v \in N$ are within each other's maximum transmitting range. Let $P_u(v)$ (respectively, $P_v(u)$) be the minimum transmit power of node $u$ (respectively, $v$) needed to sustain the link to node $v$ (respectively, $u$). Furthermore, let $N_u(v)$ (resp., $N_v(u)$) be the set of nodes within $u$'s (resp., $v$'s) transmitting range when $u$ (resp., $v$) transmits with power $P_u(v)$ (resp., $P_v(u)$). The interference number of edge $e$ is defined as $IN(e) = |N_u(v) \cup N_v(u)|$.*

The example reported in Figure 3 clarifies the definition of interference number of an edge. We believe the notion of interference number as defined in this paper is a reasonable measure of the interference generated by the communication along a certain wireless link, at least when the MAC layer is based on CSMA-CA (as it is the case of 802.11). Suppose nodes $u$ and $v$ are the communicating nodes; due to the RTS/CTS message exchange, all the nodes within $u$'s and $v$'s transmitting range (i.e., nodes in $N_u(v)$ and in $N_v(u)$) must refrain their communications to avoid interference with the current transmission. So, the number of nodes in $N_u(v) \cup N_v(u)$ (excluding the communicating nodes $u$ and $v$) is a measure of the amount of wireless medium 'consumed' by the communication.

Note that our notion of interference number can be easily extended to account for interference ranges which are larger than the communication range. This is the case, for instance, when the access to the channel is regulated by a carrier sensing mechanism, given that the carrier sensing range is usually larger than the actual transmitting range. However, to ease the presentation of our results, in the rest of this paper we assume that the interference range of a node coincides with its transmitting range.

Based on the interference number, we measure interference in a multi-hop communication as follows: given a certain path $p = \{u = w_0, w_1, \ldots, w_{h-1}, w_h = v\}$ connecting nodes $u$ and $v$, the cost of communicating from $u$ to $v$ along $p$ equals the sum of the interference numbers of the links traversed by the path. This defines the *path interference cost* of $p$. Formally:

$$PIC(p) = \sum_{i=0}^{h-1} IN((w_i, w_{i+1})) \ .$$

Given the maxpower communication graph $G = (N, E)$ and a given source/destination pair $(u, v)$ in $G$, the *minimum interference path* between $u$ and $v$ is a path in $G$ with minimum PIC, and it is denoted $mip^G_{u,v}$.

Based on the PIC, we can use the notion of spanning factor to estimate how good a certain network topology is at reducing interference:

DEFINITION 3 (PIC SPANNING FACTOR). *Let $G = (N, E)$ be the maxpower communication graph, and let $G' = (N, E')$ be a subgraph of $G$. The PIC spanning factor of $G'$ is the maximum over all possible source/destination pairs of the ratio of the cost of a minimum interference path in $G'$ to the cost of a minimum interference path in $G$. Formally,*

$$\rho(G') = \max_{u,v \in N} \frac{PIC(mip^{G'}_{u,v})}{PIC(mip^G_{u,v})} \ .$$

*Conventionally, we define $\rho(G') = \infty$ if there exist nodes $u, v$ which are connected in $G$, but they are disconnected in $G'$.*

Ideally, we want to identify a sparse subgraph $G'$ of $G$ with low PIC spanning factor, possibly equal to 1. If such a subgraph $G'$ exists, we are ensured that routing messages along $G'$ does not incur any interference penalty with respect to routing messages in the original graph. Of course, this is true under the assumption that interference-aware routing is used in combination with interference-based topology control.

With respect to this last point, we observe that the PIC can be used to implement interference-aware routing in a straightforward manner, e.g., by using the interference number as the link cost in DSR-like routing protocols [9]. We want to stress that recent work has shown that interference-aware routing has the potential to considerably increase network throughput with respect to shortest path routing (see, e.g., [3] and [8]).

The path interference cost as defined here is the first metric proposed in the literature which: (*i*) can be easily computed; (*ii*) does not require any global knowledge (e.g., number of nodes in the network) nor traffic information; (*iii*) accounts for multi-hop communications; and (*iv*) can be used in combination with transmit power control techniques.

In fact, the coverage measure proposed in [2] to estimate interference is used only to estimate the maximum possible interference experienced by links in the communication graph. Another metric for estimating interference has been proposed in [12], which accounts for the average link interference. However, *both these metrics do not account for the multi-hop nature of communications in ad hoc networks.* As we will show in the next section, not accounting for multi-hop interference leads to drawing radically different conclusions on which topologies are good for reducing interference.

Other interference metrics have been introduced in the literature. However, they either require knowledge of the traffic flows [11], or they rely on global information such as node positions, density, and expected traffic [7], or they are based on a centralized approach to the problem of reducing multi-hop interference [8]. In the context of routing, the metric that shares most properties with the path interference cost defined here is the expected transmission count metric (ETX) proposed in [3]. ETX estimates the number of transmissions required to successfully deliver a packet over a link, and it is used to find paths that minimize the expected total number of packet transmissions required to successfully deliver a packet to the final destination. Similarly to the PIC metric, ETX can be easily computed relying only on local information (link loss estimate), it does not require global information, and it accounts for multi-hop communications. Furthermore, since the number of expected transmissions is clearly related to the expected interference level in the network, ETX-based routing is likely to select low-interference paths. However, ETX relies on the assumption that all the nodes use a fixed transmit power level, and, consequently, it cannot be used in combination with topology control techniques.

# 5. TC FOR MULTI-HOP INTERFERENCE

As discussed above, the notion of interference used in the current TC literature does not account for interference in multi-hop communications. A consequence of this fact is that MST-like topologies (as they are computed by the LIFE algorithm introduced in [2], and by the various algorithms introduced in [12]) are claimed to be optimal for reducing interference. *The following theorem shows that this claim is false if the multi-hop nature of communications in ad hoc networks is accounted for.*

THEOREM 1. *Assume the nodes have a circular radio coverage area (in this case, interference number and coverage are equivalent notions). Let $G = (N, E)$ be the max-power communication graph, and assume $G$ is connected. Let $MST = (N, E_{MST})$ be a MST built on $G$ using the interference number as the edge weight. The PIC spanning factor of the MST is $\Omega(n)$, where $n = |N|$.*

PROOF. Consider the node placement reported in Figure 4. Assume the nodes' maximum transmitting range is $d$. Nodes $u$ and $v$ are at distance $d$ from each other. The remaining $n-2$ nodes form a chain, where consecutive nodes in the chain are at distance $d' < d$ from each other. Furthermore, we have the property that any two non consecutive nodes in the chain are at distance greater than $d$. The endpoints of the chain are nodes $w_1$ and $w_{n-2}$, where $w_1$ is at distance $d'$ from $u$ and at distance $> d$ from $v$, and $w_{n-2}$ is at distance $d'$ from $v$ and at distance $> d$ from $u$. Furthermore, there is a third node, $w_2$, which is at distance $d''$ from $u$, with $d' < d'' < d$, and at distance $> d$ from $v$. All the other nodes in the chain are out of $u$'s and $v$'s maximum transmitting range. The resulting communication graph is reported in Figure 4; in the figure, edges are labeled both with their length and with the interference number.

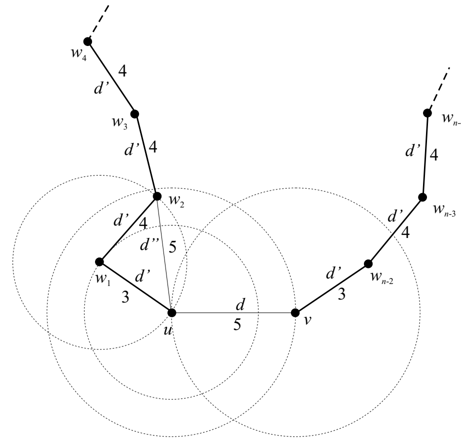


**Figure 4: Example showing that the interference-based MST has $\Omega(n)$ PIC spanning factor.**

With this node configuration, $G$ is a connected graph composed of $n+1$ edges: edges $(u, v)$ and $(u, w_2)$ have interference number equal to 5, edges $(u, w_1)$ and $(v, w_{n-2})$ have interference number equal to 3, and the remaining edges have interference number equal to 4. When computing the $MST$, all the edges of weight $< 5$ are considered before edges $(u, v)$ and $(u, w_2)$ are taken into account. Since the subgraph of $G$ obtained by considering all the edges with weight 3 and 4 is connected, it follows that links $(u, v)$ and $(u, w_2)$ are not included in the MST. The MST resulting from this node configuration is represented by bold edges in Figure 4. The minimum interference path connecting $u$ and $v$ in the MST has cost $2 \cdot 3 + 4 \cdot (n-3)$; on the other hand, the minimum interference path between $u$ and $v$ in $G$ is edge $(u, v)$, whose cost equals 5. Thus, we can conclude that the PIC spanning factor of the interference-based MST is $\Omega(n)$, and the theorem is proven. ☐

The authors of [2] introduced other low-interference topologies which, besides preserving connectivity, are good spanners. However, they consider Euclidean spanners, which in general are not good at reducing multi-hop interference.

# 6. THE ATASP TOPOLOGY

In the previous section we have proved that MST-like topologies are not appropriate for reducing multi-hop interference. What is then a good topology for this purpose? The following analysis answers this question.

DEFINITION 4 (ATASP TOPOLOGY). *Let $G = (N, E)$ be the maxpower communication graph. The (interference) ATASP subgraph of $G$ is the graph with node set $N$ and edge set $E_{ATA}$, where edge $(u, v) \in E_{ATA}$ if and only if there exists a source/destination pair $w, z$ in $N$ such that edge $(u, v)$ belongs to a minimum interference path connecting $w$ and $z$ in $G$.*

The intuition behind the notion of ATASP (All-To-All-Shortest-Path) graph is the following: in principle, an edge

$e$ can be declared "inefficient", and thus removed from the final network topology $G'$, only if it is not part of any interference-optimal path in the graph. Otherwise, removing $e$ from the network topology might increase the PIC of some optimal source/destination path, possibly leading to an increase of the PIC spanning factor of $G'$.

Note that the increase in the PIC spanning factor does not necessarily occur: in fact, it might be the case that there exist multiple minimum interference paths connecting two nodes, and removing an edge along one of these paths does not increase the PIC spanning factor. However, with the definition of ATASP graph introduced above, we are ensured that in every possible node placement ATASP has optimal PIC spanning factor. This is stated in the following theorem.

THEOREM 2. *Let G be the maxpower communication graph, and let ATASP be the graph constructed as in the definition above. ATASP has optimal PIC spanning factor, i.e.,* $\rho(ATASP) = 1$.

PROOF. The proof follows immediately by the definition of ATASP graph. □

The fact that ATASP has optimal PIC spanning factor implies that it preserves worst-case connectivity:

THEOREM 3. *Let G be the maxpower communication graph, and let ATASP be the graph constructed as in the definition above. Then ATASP is connected iff G is connected.*

Although the ATASP topology has the nice features of being an optimal PIC spanner and of preserving network connectivity, the question of whether ATASP is actually a sparse subgraph of $G$ remains open. The following theorem gives a negative answer to this question.

THEOREM 4. *There exist a node configuration and maximum transmit power setting such that the communication graph G is composed of* $\Theta(n^2)$ *edges, and its ATASP subgraph is composed of* $\Theta(n^2)$ *edges as well.*

PROOF. Consider a placement of $n$ equally spaced nodes on a circle, numbered consecutively 0 through $n-1$. Suppose also that the maximum transmitting range of a node is not less than the diameter of the circle, so that the maxpower communication graph $G = (N, E)$ is the complete graph. We prove that, for any $u, v \in N$, the arc $(u, v)$ must be in the ATASP topology, i.e., $E_{ATA} = \Theta(n^2)$.

By a trivial symmetry argument, the cost of the optimum interference path between any two nodes only depends on their *distance* $k$, measured as the minimum number of nodes between them (moving either clockwise or counterclockwise). Thus, we may assume, w.l.o.g, that $u = 0$ and $v = k$, with $0 < k \leq \frac{n-1}{2}$. A link between nodes $i < j$ is a *chord* of length $c$ if $j - i = c$. It is easy to see that $IN((i,j)) = \min\{n, 3(j-i)+1\}$. In fact, a transmission along the chord $(i,j)$ will interfere with the $c$ nodes "preceding" the sender $i$, the $c$ nodes "following" the receiver $j$, and the $c-1$ nodes in-between; counting also sender and receiver and summing up gives interference $3c+1$. See Figure 5.

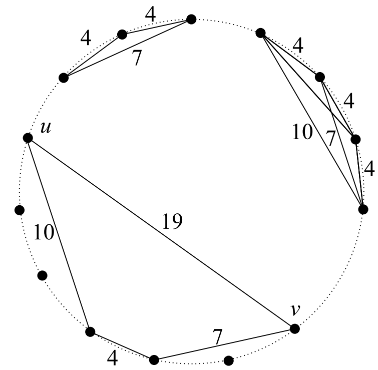


**Figure 5: Placement of nodes for Theorem 4. In this example,** $k = v - u = 6$**.**

Now, in looking for a minimum interference path between 0 and $k$, we can limit our search to monotonic paths, i.e., paths such that, for any intermediate transmission $s \to r$ (if any), $r > s$ holds true. This fact can be easily proven by induction on the value of $k$. But then, any optimum interference path $p$ between 0 and $k$ must satisfy the equation

$$\begin{aligned} PIC(p) &= \min_{\substack{0 \leq h < k \\ 0 < i_1 < \ldots < i_h < k}} PIC(0, i_1, \ldots, i_h, k) \\ &= \min_{\substack{0 \leq h < k \\ 0 < i_1 < \ldots < i_h < k}} \sum_{j=0}^{h} IN((i_j, i_{j+1})) \\ &= \min_{\substack{0 \leq h < k \\ 0 < i_1 < \ldots < i_h < k}} \sum_{j=0}^{h} (3(i_{j+1} - i_j) + 1) \end{aligned}$$

where we have set $i_0 = 0$ and $i_{h+1} = k$. The terms in the last summation telescope, giving $PIC(p) = 3k + h + 1$ which is minimized for $h = 0$, i.e., when $p$ coincides with the link $(0, k)$. Since $k$ is arbitrary, this means that *any* chord of length $k$ must indeed be present in $E_{ATA}$. □

Indeed, the very same node placement adopted in the proof of Theorem 4 can be used to prove the following stronger negative result about multi-hop interference-based TC:

COROLLARY 1. *There exist a node placement and maximum transmit power setting such that no link can be removed from the maxpower communication graph without increasing multi-hop interference.*

In words, Corollary 1 states that *there exist situations in which performing multi-hop interference-based TC is useless, since all the links in the maxpower communication graph turn out to be interference-efficient.*

Is then performing multi-hop interference-based TC pointless, as it is the case of energy-based TC? To answer this question, we first observe that Theorem 4 and Corollary 1 refer to a worst-case scenario, which is quite unlikely to occur in practical situations. To gain insights on the ATASP sparseness in average-case situations, we have estimated the average node degree in ATASP through extensive simulations on randomly deployed networks. To generate the maxpower communication graph $G$, a number $n$ of nodes is distributed uniformly at random in the unit square, and the communication graph is computed according to a certain radio channel model. Similarly to the simulations for energy, we have considered values of $n$ ranging from 10 to 500

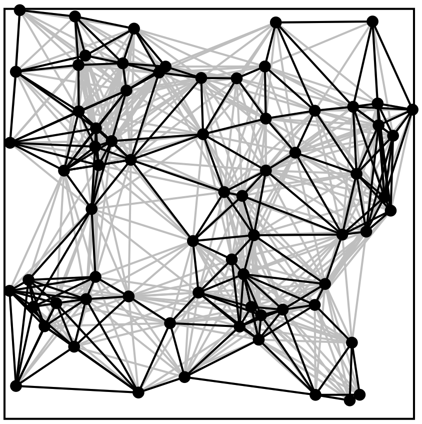

**Figure 6: Sample of ATASP graph. The radio channel model is free space.**

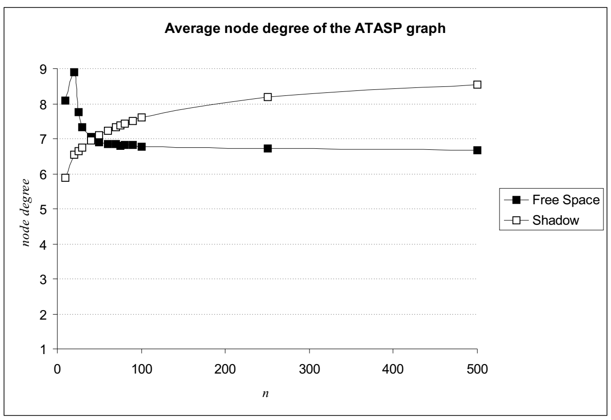


**Figure 7: Average node degree of the ATASP graph for increasing network size with free space propagation, and with log-normal shadowing ($\sigma = 6$). Node distribution is uniform.**

nodes, and two radio channel models: the quite idealistic free space propagation model, and the log-normal shadowing model.

Once the communication graph has been generated, we assign weights to the links according to the interference number, and we compute the optimal all-to-all shortest paths. Every edge which is part of at least one such paths is marked as belonging to ATASP. At the end of this process, the ATASP topology is computed, and the average node degree recorded. A sample of ATASP topology is reported in Figure 6.

The results of our simulations are reported in Figure 7. As seen from the figure, the average degree with log-normal shadowing is slightly higher than the degree with free space propagation. However, in both cases the average degree remains confined below 8.5, even for large networks.

To evaluate the effect of node concentration on the average ATASP node degree, we have repeated the simulations using the two-dimensional Normal distribution to deploy nodes. The simulation results, which are reported in Figure 8, show that the effect of node concentration on the ATASP node degree is marginal.

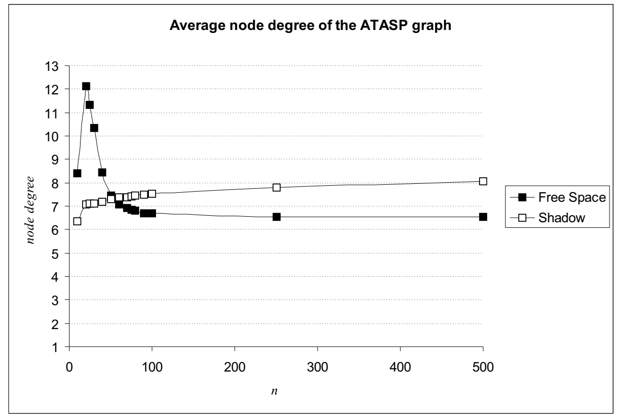


**Figure 8: Average node degree of the ATASP graph for increasing network size with free space propagation, and with log-normal shadowing ($\sigma = 6$). Node distribution is Normal.**

Overall, simulation results show that, while ATASP is a dense graph (actually, it can coincide with the maxpower communication graph) in the worst case, it is a sparse subgraph of the maxpower communication graph on the average, indicating that, if we exclude pathological node placements, multi-hop interference-based TC is actually possible.

# 7. LOCALIZED LOW-INTERFERENCE TOPOLOGIES

In the previous section we have identified ATASP as the interference-optimal topology, under the assumption that multi-hop interference is considered. Unfortunately, building the ATASP graph requires global knowledge, thus impairing one of the desired features of topology control protocols, i.e., locality.

While we leave the problem of designing a localized TC protocol for building a provably multi-hop interference optimal topology open, in this section we investigate through simulation how do existing localized topologies, which have been proposed in the literature with the purpose of reducing energy consumption (based on a quite unrealistic energy model – see Section 2), perform with respect to multi-hop interference.

The simulation setting is the same as in experiments reported in the previous sections: $n$ nodes randomly distributed in the unit square (uniform or normal distribution), values of $n$ ranging from 10 to 500, and free space or log-normal shadowing radio channel model.

We have then considered four different topologies built on the communication graph: the Relative Neighbor Graph, the Gabriel Graph, the CBTC graph [16], and the KNeigh graph [1]. For each of these graphs, we have computed the PIC spanning factor with respect to the original communication graph.

Note that, in case of log-normal shadowing propagation, we have partially modified the definitions of RNG, GG, CBTC and KNeigh graph: instead of considering the actual node distances to compute the graphs, we have considered the "virtual" distance obtained by accounting for the shadowing effect (see Section 2). For instance, in the KNeigh

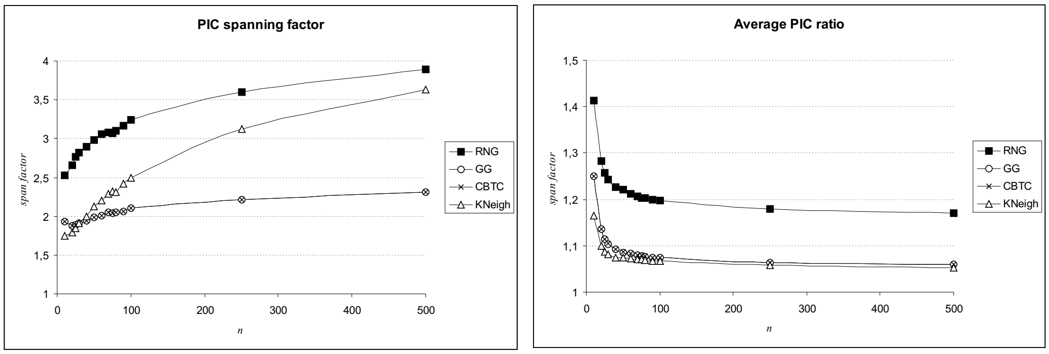


**Figure 9: PIC spanning factor (left) of different localized topologies with free space propagation. The graphic on the right show the average PIC ratio. Node distribution is uniform.**

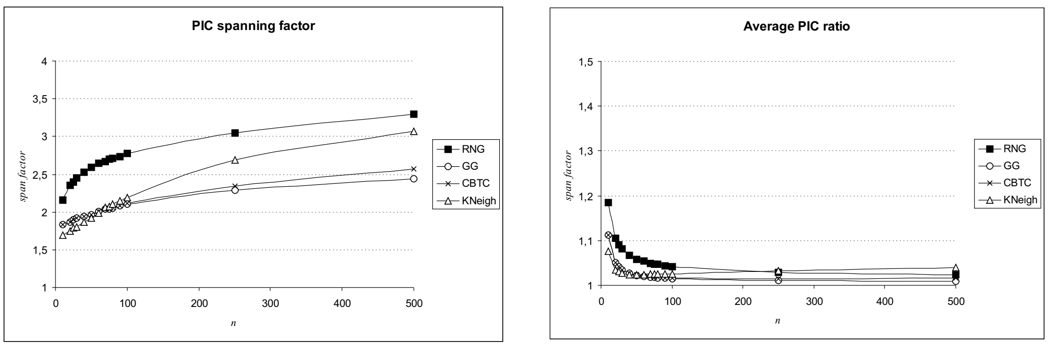


**Figure 10: PIC spanning factor (left) of different localized topologies with log-normal shadowing. The graphic on the right show the average PIC ratio. Node distribution is uniform.**

protocol, instead of connecting each node to its $k$ closest neighbors, we have connected each node to the $k$ neighbors which can be reached with the less power, independently of the actual distance to these nodes. From a worst-case perspective, this modification might cause these graphs to loose their connectivity property (we recall that RNG, GG and CBTC preserve worst-case connectivity, while KNeigh preserves connectivity with high probability). However, our simulations show that this unfortunate situation is very likely not to occur.

The results of our simulations are shown in Figure 9 for the case of free space propagation, and in Figure 10 for the case of log-normal shadowing with uniform node distribution. Besides computing the PIC spanning factor, we have also computed the *average* ratio of the cost of the interference optimal path in the topology at hand to the cost of the optimal path in $G$ (we recall that the PIC spanning factor is the maximum of these ratios). This value, which we call the average PIC ratio, gives an idea of the average interference penalty caused by using a certain subgraph of $G$ to route messages.

As seen from the figures, the PIC spanning factor of all the topologies considered remains confined below 4 in case of free space propagation, while it remains below 3.5 with log-normal shadowing. The topology that shows the best performance is the GG, with a PIC spanning factor below 2.5 with both free space propagation and log-normal shadowing. When considering the average PIC ratio, the situation is even better: the average interference penalty of the topologies considered is below 1.2 for moderate to large size networks with free space propagation, and it is below 1.1 with log-normal shadowing. The GG is the best performing topology also with respect to this metric.

The simulation results obtained with Normally distributed points, which are not reported due to lack of space, show that the effect of node concentration on the PIC spanning factor and on the average PIC ratio of the different topologies is scarcely significant.

# 8. THE TRIANGULAR INEQUALITY AND INTERFERENCE

In the previous section, we have shown that localized topologies that have been introduced in the literature with the purpose of reducing energy consumption (under an unrealistic energy model – see Section 2) turn out to perform well with respect to multi-hop interference. Among these topologies, the GG is the one that displays best performance.

In this section, we argue that this fact happens by no chance, but it is a consequence of the triangular inequality argument, which, although not valid as far as energy is concerned, turns out to hold (under certain assumptions) for multi-hop interference.

Assume that the radio coverage area is a perfect circle, and that nodes are randomly, densely distributed. In particular, we model the distribution of nodes in (2-dimensional) space according to the Poisson process with given density parameter $\lambda$. In this scenario, let $N(S)$ denote the number of nodes in the surface $S$, and let $\mu(S)$ denote the measure (i.e., area) of $S$; then

$$\text{Prob}(N(S) = k) = e^{-\lambda\mu(S)} \frac{(\lambda\mu(S))^k}{k!}. \quad (2)$$

Also, if $S_1$ and $S_2$ are disjoint surfaces, the variables $N(S_1)$ and $N(S_2)$ are independent.

THEOREM 5. *Let $u$ and $v$ be two adjacent nodes in the communication graph and let $IN((u,v))$ denote the interference number of the edge $(u,v)$. Let the nodes of the wireless network be distributed according to the Poisson process in space with density $\lambda$. Then*

$$\text{P}(IN((u,v)) = k) = \text{e}^{-\lambda\mu(S_{uv})} \frac{(\lambda\mu(S_{uv}))^k}{k!},$$

*where $S_{uv}$ is the surface depicted in Figure 2.*

PROOF. (Sketch) The result is quite intuitive. Given a distribution of nodes, we pick the edge $(u,v)$ whose interference we want to compute. The probability that the region $S_{uv}$ contains $k$ nodes "should be" the same as the probability that $S_{uv} \setminus (\{u\} \cup \{v\})$ contains $k$ nodes, since the set $\{u\} \cup \{v\}$ has measure 0. □

The actual value of $\mu(S_{uv})$ can be easily computed as twice the area of the circle of radius $r_{uv} = \text{dist}(u,v)$ minus the area $C_{uv}$ of the intersection of two such circles whose centers are at distance $r_{uv}$ (see Figure 2). Because of the symmetry, the latter can be computed as follows:

$$C_{uv} = 4 \cdot \int_0^{\frac{\sqrt{3}}{2} r_{uv}} \left( \sqrt{r_{uv}^2 - x^2} - \frac{1}{2} r_{uv} \right) \text{d}x = \\ = \left( \frac{2}{3}\pi - \frac{\sqrt{3}}{2} \right) r_{uv}^2 \ .$$

Twice the area of the circle minus the above value gives then

$$\mu(S_{uv}) = 2\pi r_{uv}^2 - C_{uv} = \left( \frac{4}{3}\pi + \frac{\sqrt{3}}{2} \right) r_{uv}^2 = \gamma r_{uv}^2 \,,$$

where $\gamma \approx 5.0548$.

Given two adjacent nodes $u$ and $v$, it is not easy to compute the probability of the following event: the interference over $(u,v)$ is smaller (larger) than the sum of the interferences over the edges $(u,w)$ and $(w,v)$, where $w$ is a third node adjacent to both $u$ and $v$. In fact, the events "number of nodes in $S_{xy}$" (where $x, y \in \{u,v,w\}$ and $x \neq y$) are highly dependent.

On the side of expectations, though, the computation is straightforward. In fact, we have

$$\begin{aligned} \text{E}[N_{uw} + N_{wv}] &= \text{E}[N_{uw}] + \text{E}[N_{wv}] = \\ \lambda\mu(S_{uw}) + \lambda\mu(S_{wv}) &= \lambda\gamma(r_{uw}^2 + r_{wv}^2) \end{aligned}$$

Analogously, $\text{E}[N_{uv}] = \lambda\gamma r_{uv}^2$, and thus $\text{E}[N_{uw} + N_{wv}] \leq \text{E}[N_{uv}]$ if and only if $r_{uw}^2 + r_{wv}^2 \leq r_{uv}^2$. This amounts to saying that $w$ must not lay within the circle having the edge $(u,v)$ as the diameter. This corresponds exactly to the definition of Gabriel Graph.

# 9. CONCLUSIONS

In this paper we have demonstrated the importance of accurately choosing the energy and interference model when studying the topology control problem in wireless ad hoc networks. While we do not promote ours as the best possible energy and multi-hop interference models for ad hoc networks, we believe that they capture the features of this type of networks better than the models used in the literature so far. As a consequence, we believe the conclusions about TC presented in this paper are closer to reality than the ones presented in previous work. We are currently working on setting up an experimental testbed for some of the TC techniques considered in this paper, in order to experimentally validate our findings. This testbed could also be used to demonstrate the capability of topology control to increase network throughput in a realistic setting.

While this and other recent papers represent progress on the topic of minimizing interference in ad hoc networks, much work remains to be done on this topic. There are also many interesting open questions surrounding the interplay between interference, energy, delay, and throughput.

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