

A Two-Dimensional Geometry-based Stochastic Model

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Abstract

In this paper, the problem of deriving the power angular scattering response (PASR) resulting from a Gaussian scatterer distribution is undertaken. Differently from previous work, the scatter cluster can be located arbitrarily in the vicinity of the mobile unit. Furthermore, for the considered scatter geometry we derive both the angular and distance statistics at the observation point, allowing for the first time a theoretical characterization of PASR. A major byproduct of this analysis is that the Gaussian scatterer hypothesis is formally shown to produce a Gaussian power angular spectrum in a macrocell environment, which is in accordance with existing measurements.

Index Terms

Angular power scattering response, wireless spatial channel modeling, antenna arrays.

I. INTRODUCTION

COMMUNICATION architectures that employ multi-element antennas have gained considerable attention in the academic community in recent years. In particular, the considerable throughput capacity benefits potentially offered by these systems have increased interest in spatial channel modeling. In fact, spatial channel modeling is fundamental in estimating one of the key performance indicators in multi-element antennas, i.e. the correlation experienced between adjacent links which, in turn, depends on the response of each antennae in the array. Therefore, understanding the origins of correlation may help us to resolve some of today's intriguing tasks in antenna design and pattern diversity. Correlation is largely affected by the so-called angular power spectrum (APS), also known as power azimuth spectrum (PAS) in the two-dimensional (2-D) plane. The PAS has been extensively used for the estimation of channel statistics in various standardized spatial channel models, e.g. [1], [2].

In the 2-D case, the authors in [3] observed through experimental investigations that a Gaussian distribution in angle of arrival (AoA) gives rise to a Laplacian-like PAS. The Laplacian function was also considered in [4] for modeling the AoA of multipaths, which may be attributed to the congested environment in this case. In general, many different propositions of AoA and APS have been made considering different scatterer distributions such as Gaussian, von Mises and uniform. For instance, in

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[5] Janaswamy assumed a Gaussian scatter density around the mobile station (MS), and derived the corresponding AoA and APS as seen by the base station (BS). Joint time of arrival and AoA statistics were also derived in [6] assuming uniform distribution of scatterers. Therefore, in [5], [6] the authors derived the spatial channel statistics in accordance with a given geometrical arrangement of scatterers. However, what is lacking in both approaches is a functional characterization of the angular *power* spectrum, while attention was concentrated towards the joint angle and time of arrival statistics.

The main goal of this paper is to present a theoretical framework for a *complete* (i.e., including PAS) 2-D spatial channel characterization under the Gaussian scatterer assumption¹. As discussed in the next section, a prerequisite for a complete spatial channel characterization is modeling the overall scattering response in 2-D as a function of both *distance* and *angle* to the observation point. Starting from a Gaussian distribution of scatterers in two dimensions around a random point in space (observation point- ρ_1), the corresponding distance-dependent AoA statistics are derived. The term distance angular scattering response (DASR) is introduced to refer to the derived joint probability density function (pdf) of AoA and distance. By turning distances into powers using a conventional path loss model, the desired 2-D power angular scattering response (PASR) is then derived.

This paper improves the state-of-the-art spatial channel modeling approaches under several respects. First, to the best of our knowledge the proposed is the first theoretical model jointly considering distance and angular statistics, thus allowing for the first time a theoretical characterization of the expected PAS. Second, it allows scatterers to be centered around a random point in the plane, instead of around the MS as in [5]. Finally, the study presented in this paper provides for the first time a theoretical backup to the fact that the PAS observed at the MS has a Gaussian shape in a macrocell scenario, which at least partially matches with existing measurement-based studies [7], [8].

Applications of this work include the simulation of the antenna's response under a realistic and well-defined power angular field, and antenna response optimization in order to minimize the correlation experienced between the elements (hence increasing link capacity).

II. 2-D SCATTER MODEL

The geometric model of reference is reported in Figure 1: a cluster of scatterers is randomly distributed around a point in the plane – named the *center of gravity* (CoG for short) in the following – according to a bi-variate Normal distribution. A receiver node Rx is located in another point of the plane, assumed to

¹The Gaussian scatterer assumption is motivated by its emergence in experimental studies [3].

be the origin of the Euclidean plane. Deterministic elements in the geometry are the position $Rx = (0, 0)$ of the receiver, and the position $CoG = (x_0, y_0)$ of the CoG. Random elements in the geometry are the positions of specific scatterers, represented as gray circles in Figure 1. Note that, differently from [5], the scatter CoG is not assumed to be co-located with the Rx; instead, the CoG is located at an arbitrary position (x_0, y_0) in the plane, as defined by vector Ω_o reported in Figure 1.

It is of interest in this letter to derive a model predicting the PASR observed at Rx. It is important to note that the derived expression is a distribution and not a spectrum (PAS). In other words, the derived PASR allows determining, for each incoming direction φ and possible power value P , the probability density of observing exactly power P incoming from direction φ at a generic instant of time t . The PAS can instead be intended as the average amount of power incoming from a specific direction, where averaging is done in the temporal dimension. Turning the PASR derived herein into the traditional notion of PAS is then an exercise amounting to derive, for each given direction φ , the *expected* amount of power incoming from direction φ . In mathematical terms the above description is equivalent to: $PAS = E[f(\varrho_r|\varphi)]$, where ϱ_r defines the received power along direction φ .

Note that two random quantities need to be derived in order to estimate the PASR: fixed an azimuth direction φ , what needs to be characterized is the density of scatterers observed by Rx along direction φ – i.e., the angular scatterer density –, and the density of power received at Rx incoming from a scatterer along direction φ – i.e., the scatterer power density (see Figure 1). The composition of these two densities allows the derivation of the desired PASR.

It is important to observe that, in order to estimate the scatterer power density – i.e., the pdf of the random variable $P_{Rx,\varphi}$ denoting the power received at Rx incoming from a scatterer along direction φ –, two quantities need to be derived: *i*) the pdf of the *distance* between Rx and a scatterer along direction φ (denoted r_1 in Figure 1), and *ii*) the pdf of the *distance* between a scatterer along direction φ and the transmitter Tx (denoted ρ_1 in Figure 1). In fact, if *i*) and *ii*) are known, conventional path loss models can be used to convert *ii*) into an estimate of the amount of power emitted by a scatterer as a response to transmission from Tx, and to convert *i*) into an estimate of how much of this power is received at Rx.

Note that, while distance distribution *i*) is relatively easy to obtain since the distance $\|\Omega_o\|$ between Rx and the CoG is a deterministic element of the reference geometry, deriving *ii*) is relatively more complex. Two approaches can be undertaken here: in the first approach, the position of transmitter Tx is not known. Our interest in this case is deriving the PASR observed at Rx conditioned on a specific distance d between

Tx and CoG. In this situation, *ii*) can be derived along the same lines as *i*) by simply substituting $\|\Omega_o\|$ with d . In other words, lengths ρ_1 and ρ_2 in Figure 1 can be considered as independent random variables. In the second approach, the position of transmitter Tx is known, hence vectors $\|\Omega_c\|$ and $\|\Omega_d\|$ are two other deterministic elements in the geometry. The analysis of this case is more complex than the previous one, since now lengths ρ_1 and ρ_2 are correlated random variables. More specifically, given ρ_1 , the value of ρ_2 can be computed applying the law of cosines to triangle (Rx,Tx,S), where angle γ is a random variable introducing a non-trivial correlation between ρ_1 and ρ_2 . Although we have been able to analyze both cases, in this letter the analysis referring to the model in which the position of transmitter is unknown is presented. This analysis is more intuitive and relatively more tractable.

Fig. 1 HERE

III. TRANSFORMATION OF THE 2-D GAUSSIAN SCATTER DENSITY

In this section, a derivation of the DASR under a 2-D Gaussian scatter function is detailed under the pre-described geometry. Assume that the scatterers are distributed around an observation point $Rx = \rho_1 = (0, 0)$ in x, y under the constrained Gaussian distribution, with mean $\mu = (x_0, y_0)$ and standard deviation σ . Independence is preserved, allowing the joint density function $f(x, y)$ to be written as the product of the individual marginal densities $f(x)$ and $f(y)$. Resultantly, the distance of each directional vector to a scatterer $sc = (x, y)$ is given by $\|\Omega_{\rho_1, sc}\| = (x^2 + y^2)^{1/2}$, and its associated orientation by an azimuthal angle $\varphi = \arctan(y/x)$. To proceed, let the Gaussian density function for a random variable X be expressed as follows:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-x_0)/2\sigma^2}, -\infty \leq x \leq \infty, \quad (1)$$

which suggests that the product of the marginal densities in accordance with (1) results in

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} e^{-[(x-x_0)^2+(y-y_0)^2]/2\sigma^2}. \quad (2)$$

This function strictly characterizes the scatterer locations in space, hence not revealing any angular information at Rx. The concentration of scatterers around the mean vector (x_0, y_0) is defined through the parameter σ (standard deviation). Transforming into the more convenient spherical co-ordinate system allows us to view this problem from the angular domain. After simple algebraic manipulation and considering the Jacobian of the transformation, the joint density function with respect to the new co-

ordinate system becomes [9]

$$f_{R,\Phi}(\|\Omega_{\rho_1,sc}\|, \varphi) = \frac{\|\Omega_{\rho_1,sc}\|}{2\pi\sigma^2} e^{-(\|\Omega_{\rho_1,sc}\|^2 + \|\Omega_o\|^2)/2\sigma^2} e^{\|\Omega_{\rho_1,sc}\|\|\Omega_o\|\cos(\varphi-\varphi_o)/\sigma^2}. \quad (3)$$

Note that $\|\Omega_o\|$ denotes the distance of the mean directional vector Ω_o (recall Figure 1), and φ_o is the associated azimuthal angle to this vector. This function represents the so-called distance-dependent AoA spectrum as observed at $Rx = \rho_1$. Under this representation, the DASR at ρ_1 is taken with respect to the center of gravity (x_o, y_o) of the scatterers in its vicinity.

IV. ANALYSIS OF THE DISTANCE DISTRIBUTION

To investigate the properties of the joint distribution function, a decomposition into the distance and angular domains is required. To derive the distribution of distances $f(\|\Omega_{\rho_1,sc}\|)$, ignoring any constraints imposed by the amplitude of Ω_o , equation (3) is integrated with respect to the angular domain:

$$\begin{aligned} f(\|\Omega_{\rho_1,sc}\|; \|\Omega_o\|, \sigma) &= \int_0^{2\pi} f(\|\Omega_{\rho_1,sc}\|, \varphi) d\varphi \\ &= \frac{\|\Omega_{\rho_1,sc}\|}{2\pi\sigma^2} e^{-(\|\Omega_{\rho_1,sc}\|^2 + \|\Omega_o\|^2)/2\sigma^2} \int_0^{2\pi} e^{\|\Omega_{\rho_1,sc}\|\|\Omega_o\|\cos(\varphi-\varphi_o)/2\sigma^2} d\varphi \\ &= \frac{\|\Omega_{\rho_1,sc}\|}{\sigma^2} e^{-(\|\Omega_o\|^2 + \|\Omega_{\rho_1,sc}\|^2)/2\sigma^2} I_0\left(\frac{\|\Omega_o\|\|\Omega_{\rho_1,sc}\|}{\sigma^2}\right) \geq 0, \end{aligned} \quad (4)$$

where $\|\Omega_o\|$ is a shape parameter, σ controls the width of the distribution. The function $I_0(n)$ denotes the Modified Bessel function of the first kind and zero-integer order [10]. An evaluation of the derived distribution in (4) reveals two types of behavior depending on the ratio $\|\Omega_o\|/\sigma$. Figure 2 shows two cases where the magnitude of Ω_o gradually increases. Observe that the distribution of distances obtains a Gaussian-like shape for large $\|\Omega_o\|/\sigma$. As shown, in the lower mean distance region, the distribution is asymmetrical to the mean; evident from its left-skewness. To understand the exact relationship between the derived distribution in (4) for the limiting cases of small and large mean distances a goodness-of-fit assesment was performed. One million pseudo-random samples were generated using the rejection sampling technique. The scenarios investigated were for a large and a small mean distance respectively. It was observed that for small mean distances the Nakagami distribution provides an excellent fit to the data, while for larger mean distances the Gaussian distribution is more appropriate. It is important to note that the transition between the Nakagami and Gaussian regime of the distance distribution occurs when $\|\Omega_o\|$ is about 2σ . The distribution of distances will from this point onwards be treated as a two-folded distribution that obtains a symmetric and asymmetric shape, which only depends on the position of the

observation point relative to the cluster. It is important to observe that the transition between the Nakagami and Gaussian regime of the distance distribution occurs when $\|\Omega_\rho\|$ is about 2σ . The division into the symmetric and asymmetric regimes facilitates the derivation of a compact closed-form solution for the power angular scattering response.

Fig. 2 HERE

V. 2-D ANGULAR SCATTERING RESPONSE

In this section the angular distribution of incoming waves under the 2-D Gaussian scatter density model is examined. The derived distribution expresses the true angular scattering response prior the introduction of any power dependency. As shown in the following, the concentration of angles increases as the length of the mean distance vector $\|\Omega_o\|$ increases. Further, one may show that the distribution of angles will always follow the von Mises distribution by appropriate conditioning of each scatterer's distance to the observation point ρ_1 . The general form of the distribution of angles is obtained by integrating (3) over all possible distances:

$$\begin{aligned} f(\varphi; \|\Omega_o\|, \sigma) &= \int_0^\infty f(\|\Omega_{\rho_1,sc}\|, \varphi) d\|\Omega_{\rho_1,sc}\| \\ &= \frac{1}{4\pi} e^{-\|\Omega_o\|^2 \sin[\phi - \varphi_o]^2 / 2\sigma^2} \left(2e^{-\|\Omega_o\|^2 \cos[\phi - \varphi_o]^2 / 2\sigma^2} \right. \\ &\quad \left. + \frac{1}{\sigma} \|\Omega_o\| \sqrt{2\pi} \cos[\phi - \varphi_o] \left(1 + \text{Erf} \left[\frac{\|\Omega_o\| \cos[\phi - \varphi_o]}{\sqrt{2}\sigma} \right] \right) \right). \end{aligned} \quad (5)$$

This is the general form of the distribution of angles for a Gaussian distribution of scatterers in 2-D. The distribution of angles is a function of φ . It can be shown that increments in the length of Ω_o cause an increase in the concentration of angles. Hence, the desired effect of increased distance from the observation point is included in the proposed 2-D scattering model. The functional form of (5) is not shown in this work due to limited space.

VI. ANGULAR SCATTERING RESPONSE WITH PATH POWERS

A. Transforming distance into power

In this section the concept of a power based scattering response is developed. Our motivation originates from the fact that the information captured by the amplitudes of the vectors in the distribution of distances, does not reveal the true power received along them. In the following, the distribution of distances near to the mobile ρ_1 is assumed to be Nakagami $\sim (\mu', \omega')$, which is representative of a macrocellular scenario with the scatterers being relatively close to the receiver as opposed to the transmitter. A hypothetical

transmitter is placed in the direction of vector Ω_d , whose length determines the separation distance between transmitter and receiver. The length of the mean distance vector $\Omega_d \gg \Omega_o$. The power extracted at the scatter cluster is equivalent to

$$\varrho_e = \alpha / \|\Omega_{\rho_2,sc}\|^2, \quad (6)$$

and sets the basis of our transformation function, with α accounting for transmit power, gain of the antenna, scattering cross sectional area. All subsequent computations have been performed assuming $\alpha = 1$, however this value can be changed once our knowledge of the previously mentioned parameters has been updated. In the above formula, ρ_2 denotes the (random) position of the transmitter Tx, and, under the first of the two approaches mentioned in Section II, the distribution of random variable $\|\Omega_{\rho_2,sc}\|$ is equivalent to distribution of random variable $\|\Omega_{\rho_1,sc}\|$ defined in Section IV. However, due to the large distance between transmitter and scatterer cluster assumed in the macrocell scenario at hand, random variable $\|\Omega_{\rho_2,sc}\|$ is within the region where the Gaussian distribution of distances provides a good fit (recall discussion at the end of Section II). Let us assume that the parameters of the Gaussian distribution are $\|\Omega_{\rho_2,sc}\| \sim N(\Omega_c = 10, \sigma = 3)$. Note that random variable $\|\Omega_{\rho_2,sc}\|$ in the denominator of (6) is raised to the power of two since a free space path loss model has been assumed.

Equation (6) advises us that the power diminishes with the square of the distance (in its simplest form). It is trivial to show that if $\|\Omega_{\rho_2,sc}\|$ is a Gaussian random variable then $W = \|\Omega_{\rho_2,sc}\|^2$ has the density function given by (7) that is akin in functional form to the well known chi-square distribution with one degree of freedom ($k = 1$). The full derivation is omitted in here, while simply expressing the squared distances from the transmitter to the scatter cluster by the following PDF:

$$f_W(w) = \frac{1}{2C\sqrt{2\pi}\sigma\sqrt{w}} e^{-(\sqrt{w}-\mu)^2/2\sigma^2}, \quad (7)$$

where C is a normalization constant given by, $C = 1/2 (1 + \text{Erf} [\mu/\sigma\sqrt{2}])$. Subsequently, to transform the distance distribution into a power distribution extracted at the scatter cluster we re-write (6) as follows:

$$\varrho_e = \frac{\alpha}{w} \Rightarrow w = \frac{\alpha}{\varrho_e}, \quad (8)$$

whose derivative with respect to the variable ϱ_e is given by

$$\frac{dw}{d\varrho_e} = -\frac{\alpha}{\varrho_e^2}. \quad (9)$$

After substitution of (8) into (7) and making use of the above derivative, the transformed power with

respect to $\|\Omega_{\rho_2,sc}\|^2$ at the scatter cluster is obtained:

$$f(\varrho_e) = f_W(w) \left| \frac{dw}{d\varrho_e} \right| = \frac{\alpha}{\varrho_e^2 (C) 2\sqrt{2\pi} \sqrt{\alpha/\varrho_e} \sigma} e^{-\left(\sqrt{\alpha/\varrho_e} - \mu\right)^2 / 2\sigma^2}. \quad (10)$$

The evaluation of the above probability density function for various values of ϱ_e results in Fig. 3. The distribution of P_e behaves identically to the Inverse-Gamma (IG), which may be shown by taking a random sample from (10) and measuring its fit using the Kolmogorov-Smirnov test. Due to the very accurate fit provided by the IG in this instance, the distribution in (10) whose density form is not known to the authors will be approximated with an $IG \sim (\alpha_2, \beta_2)$. The fit becomes evident in Fig. 3.

Fig. 3 HERE

Following the same guideline, a derivation for the power received at the receiving unit is now provided. The power received P_r may be expressed as a function of the distances from the scatter cluster to the receiver using the following relationship:

$$P_r = \beta \frac{P_e}{Y}. \quad (11)$$

The constant β typically accounts for the antenna gain and effective aperture. Random variable $Y = \|\Omega_{\rho_1,sc}\|^2$ in the above formula corresponds to the squared distance between a scatterer and Rx, again motivated by the free space path loss assumption. Herein, it is known that P_e follows the distribution in (10), and that the denominator is distributed under the Gamma model with parameters ($k = \mu'$, $\theta = \omega'/\mu'$). The ratio of these two random variables results in the third random variable of interest. The problem of estimating the distribution of the ratio P_e/Y may be approached by recognizing that $P_e/Y = P_e \times (1/Y)$. The $1/Y$ ratio follows an inverse Gamma distribution with known parameters, i.e. $1/Y \equiv \Xi \sim IG(\alpha_1 = k, \beta_1 = \theta^{-1})$. This is a notable simplification, since now the ratio corresponds to the product of two inverse gamma random variables. To begin, let $\Psi = P_e \Xi$ and introduce a new random variable $\Phi = \Xi$. The joint density may be expressed as follows:

$$f_{\Psi,\Phi}(\psi, \phi) = f_{P_e}(\varrho_e) f_{\Xi}(\xi)/\phi = f_{P_e}(\psi/\phi) f_{\Xi}(\phi)/\phi. \quad (12)$$

The above function after integration directly leads to the desired marginal density:

$$\begin{aligned} f_{\Psi}(\psi) &= \int_0^{\infty} \frac{1}{\phi} f_{P_e}(\psi/\phi) f_{\Xi}(\phi) d\phi = \int_0^{\infty} \frac{1}{\phi} \left(\frac{\beta_1^{\alpha_1} \beta_2^{\alpha_2}}{\Gamma[\alpha_1] \Gamma[\alpha_2]} \left(\frac{\psi}{\phi}\right)^{-\alpha_1-1} e^{-\frac{\beta_1 \phi}{\psi}} \phi^{-\alpha_2-1} e^{-\frac{\beta_2}{\phi}} \right) d\phi \\ &= 2\beta_1^{\alpha_1} \beta_2^{\alpha_2} \psi^{-1-\alpha_1} \beta_2^{\alpha_1-\alpha_2/2} \left(\frac{\beta_1}{\psi}\right)^{\frac{-\alpha_1+\alpha_2}{2}} \mathbf{K}_{-\alpha_1+\alpha_2} \left(2\sqrt{\beta_2} \sqrt{\beta_1/\psi}\right), \end{aligned} \quad (13)$$

where $\psi = \varrho_r$, and K denotes the modified Bessel function of the second kind and order $(-\alpha_1 + \alpha_2)$. Therefore, the product of two inverse gamma random variables produces another inverse gamma random variable with different parameters. The evaluation of (13) appears in Fig. 4. The fit of the derived power distribution in a practical scenario becomes evident by comparison of the angular delay histogram derived in [3]. To conclude it is this density jointly with the angular density exhibited at any point in space that should form the so-called power angular scattering response. To complicate things further, in a clustered environment the numbers of spatial clusters need not be one, where it becomes necessary to model the spatial domain as a mixture of distributions with a power prior weight defining each components contribution to the overall power received. The existence of multiple scatter clusters has been confirmed in the literature [11], [12]. Procedures for estimating the parameters of various clusters identified in measurements can be found in [13], among others. The multimodal power angular scattering response will be developed in future work. In the last section of this work the joint PASR is briefly analyzed, where it is shown that a Gaussian distribution of scatterers in space sets the foundations for a Gaussian angular power density function.

Fig. 4 HERE

B. Power angular scattering response

To continue along the research lines of this section, it should be noted that the joint spectrum in (3) may be written independently in terms of distances and angles. Hence, transforming the distance distribution into a power distribution in each differential angular element allows the derivation of the Gaussian-based 2-D PASR sought in this work. Accordingly, the received PASR may be expressed by the following product:

$$f(\varrho_r, \varphi) = f(\varrho_r) \times f(\varphi; \|\Omega_o\|, \sigma) \quad (14)$$

The logarithmic scale in dB of the corresponding spectrum is depicted in Fig. 5. A Gaussian scatter distribution in space gives rise to a Gaussian angle of arrival, an inverse Gamma power distribution and a Gaussian angular power density function. Note that the theoretical results presented in this work cover the whole azimuthal range. The results are comparable to the findings of [7], although in the corresponding reference the authors present the results in a spectrum form and not a density function. As commented in Section II, the PASR derived herein can be turned into a PAS by taking the expectation of the conditional

distribution of power in each angle. The derivation (not shown due to lack of space) reveals that the PAS follows a Gaussian distribution.

Fig. 5 HERE

VII. WORK SUMMARY

In this paper, a novel 2-D geometry-based stochastic model has been developed. Derivations are presented for both the angular and distance domains. As shown, a Gaussian scatterer distribution in space generates a Gaussian angular power density function. The proposed model shows good agreement with other measurement-based results found in the literature, in terms of the derived angular and power density functions. The target of this work has been the construction of a 2-D angular power scattering response including the power-distance dependency. Future work entails the estimation of the correlation matrix under the resultant density. This application assists in estimating the degree of pattern diversity achieved in terms of the correlation experienced between any two given links.

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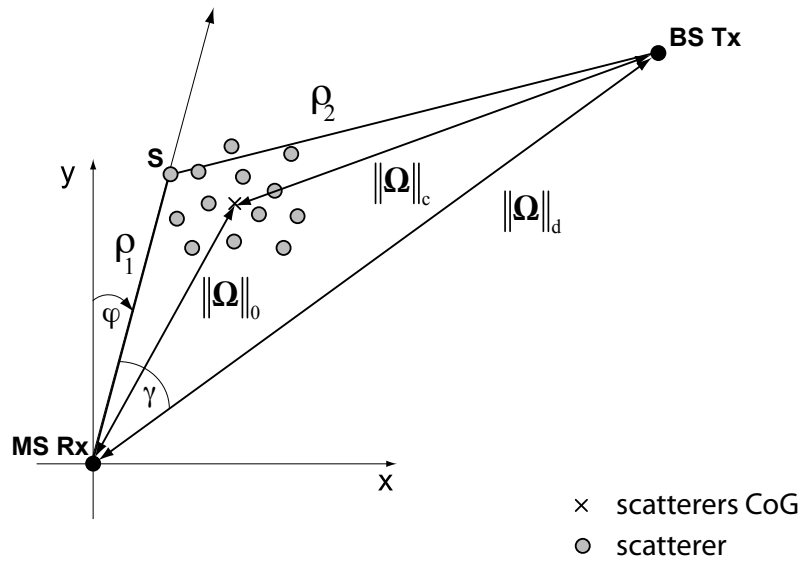
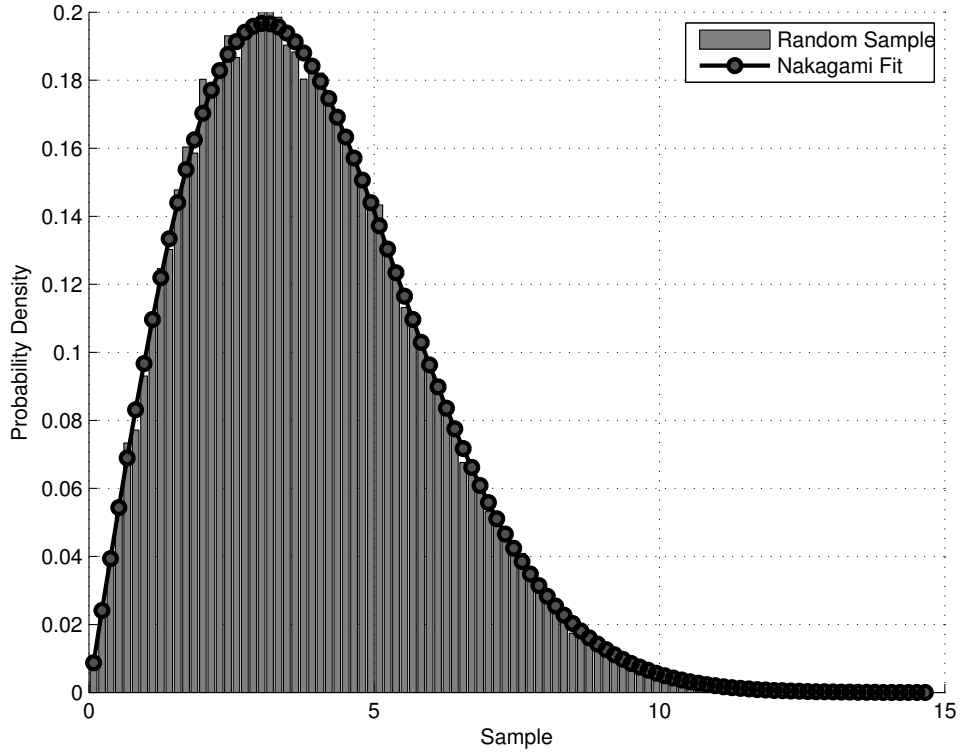
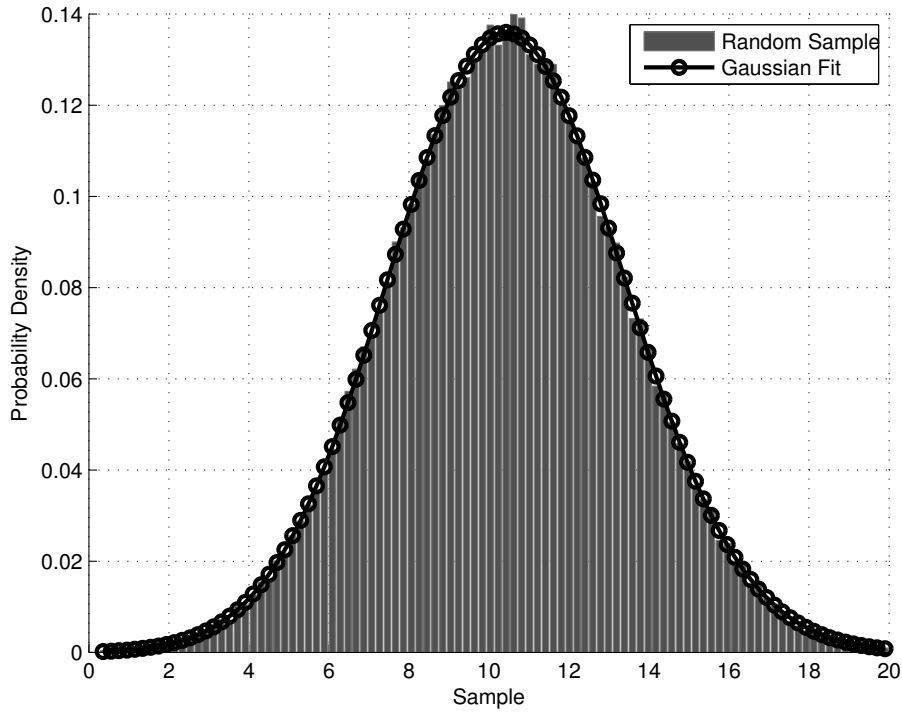


Fig. 1: This figure shows the geometrical arrangement.



(a) $\|\Omega_o\| = 1$.



(b) $\|\Omega_o\| = 10$.

Fig. 2: This figure shows the goodness-of-fit assessment by evaluating the distance pdf in (4) for various values of the mean distance vector. As shown, when $\|\Omega_o\|$ is large the distribution becomes Gaussian-like in shape, while for smaller $\|\Omega_o\|$ values a left-skewed shape is exhibited.

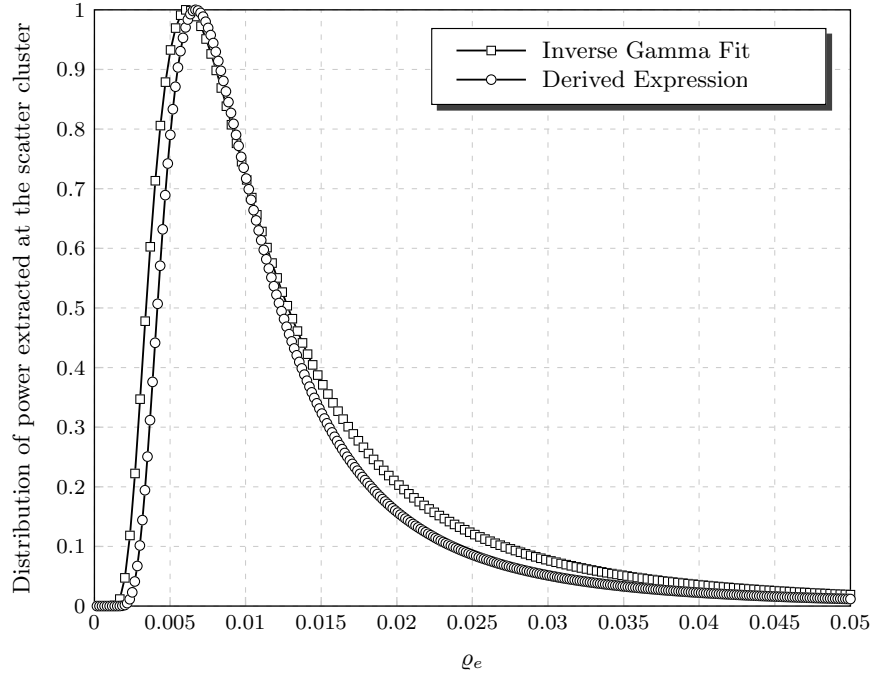


Fig. 3: This figure illustrates the probability density of power $f(\rho_e)$ as dictated by $f_W(w)$ at the scatter cluster accompanied by the goodness of fit of the Inverse-Gamma distribution (with parameters $\alpha_2 = 2.25, \beta_2 = 0.02$) to the derived PDF in (10). The Kolmogorov-Smirnov test statistic was not rejected at 5% significance level.

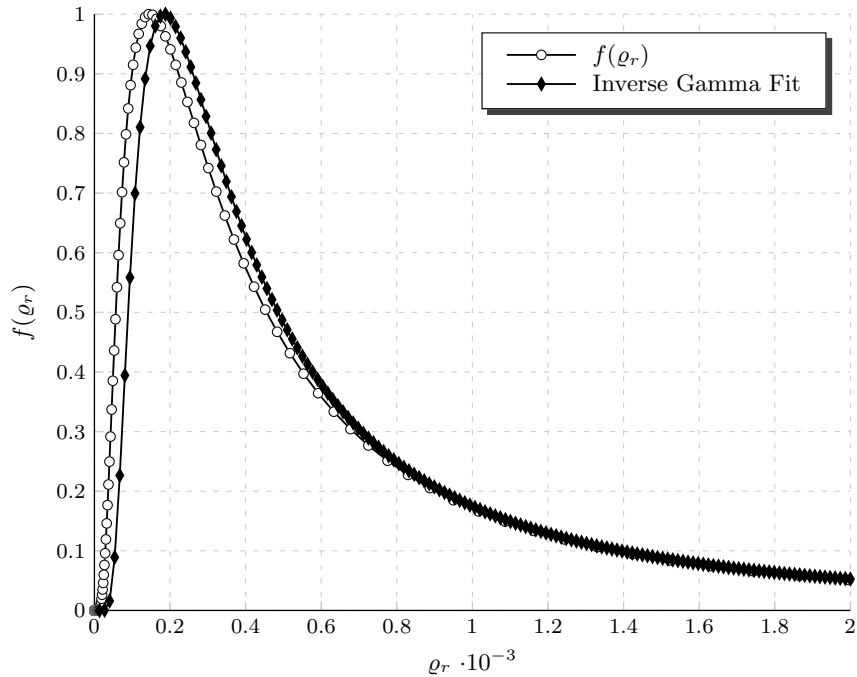
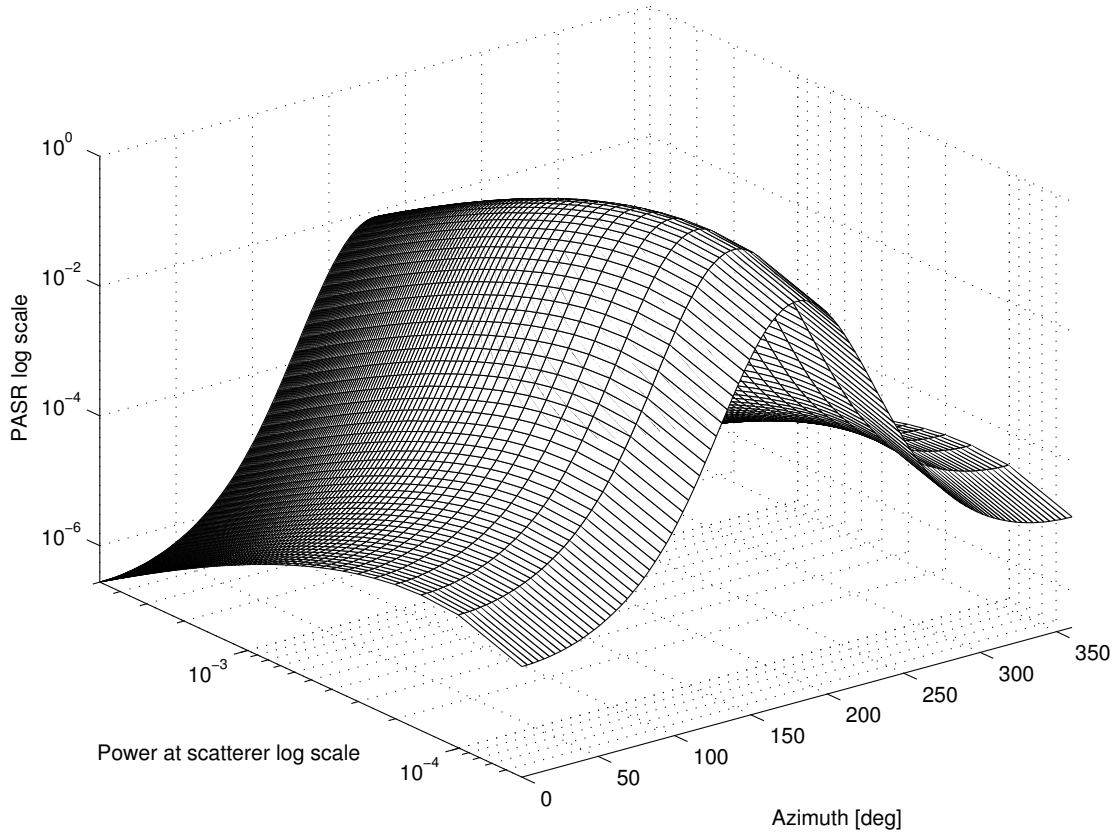
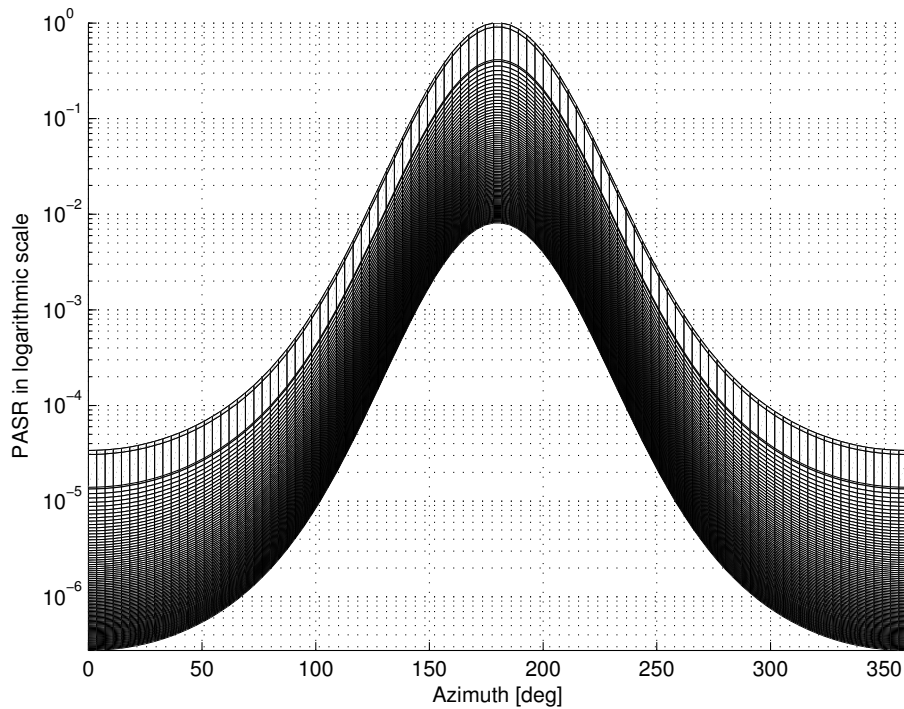


Fig. 4: Evaluation of the derived power density function at the receiver. The distribution behaves identically to the inverse gamma distribution with estimated parameters $\alpha_3 = 1, \beta_3 = 0.00037$.



(a) Logarithmic scale of the power angular scattering response in (14).



(b) Azimuthal view of spectrum.

Fig. 5: The two figures provided show sequentially the angular power scattering response of the Gaussian 2-D model. The azimuthal view suggests a very good fit between the derived model and the model fit through measurements in [3], [7].