Latency and Capacity Optimal Broadcasting in Wireless Multihop Networks

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Abstract— **In this paper, we study the fundamental properties of broadcasting in multi-hop wireless networks. Previous studies have shown that, as long as broadcast capacity is concerned, asymptotically optimal broadcasting is possible in wireless multi-hop networks under very general conditions. However, none of the existing work on broadcast capacity has considered *latency* in message delivery, which is simply assumed to be finite (but not explicitly bounded). In this paper, we address the issue of investigating the fundamental properties of broadcast communications for what concerns *both* capacity *and* latency using a realistic, SINR-based interference model. In particular, we introduce a novel topological notion of network connectivity, and show that, if the network satisfies this property, asymptotically optimal broadcast capacity and latency *can be achieved simultaneously*. This is in sharp contrast to similar results obtained for the case of unicast transmissions, where strictly bounded latency in message delivery can be achieved only at the expense of asymptotically reducing network capacity. Thus, the results presented in this paper show that scalable broadcasting in multi-hop wireless networks is, in principle, possible.**

I. Introduction

Investigation of fundamental properties of wireless multi-hop networks has received considerable attention in the research community, since it can help understanding what can and cannot be done in such networks. In their seminal work [7], Gupta and Kumar investigate the asymptotic capacity of a wireless multi-hop network for unicast transmissions, and show that network capacity *does not scale*: as the number n of network nodes increases, the per-node available capacity *decreases* as $O(1/\sqrt{n})$ in arbitrary networks, and as $O(1/\sqrt{n \log n})$ in random networks. This lack of scaling of network capacity is due to the relay burden on intermediate nodes caused by the multi-hop nature of communications: i.e., due to limited transmission range, a packet must travel several hops in order to reach the destination, and thus a single transmission results in a series of relay re-transmissions interfering both with other packets on the same flow, and with packets on different flows. For this reason, we say that unicast transmissions in a wireless multi-hop network are *relay limited*. However, if unbounded delays on message delivery can be tolerated, optimal network capacity scaling (i.e., $\Theta(W)$ per-node capacity, where W is the channel capacity) is achievable at least in mobile environments [6], under the additional assumption of unlimited buffer capacity on the nodes.

Since these seminal works, many authors have tried to gain a better understanding of the fundamental tradeoffs between network capacity and latency in packet delivery [1], [10], [14], [15]. For instance, in [10] Neely and Modiano proved that $\lambda \leq \Theta(\frac{D}{n})$, where λ is the average per-node throughput and D is the average packet delay, which implies that $\Omega(W)$ per-node throughput can be achieved only by allowing relatively high ($O(n)$) packet delay. More recently, Ozgur et al. [11] argued that $\Omega(W)$ per-node capacity can indeed be achieved also in a static network with strictly bounded latency using cooperative communication. However, this result is quite controversial, since it implicitly assumes availability of an arbitrarily large number of independent information channels between group of nodes. Franceschetti et al. [4] have shown that the actual number of independent channels across two regions in a two-dimensional domain is indeed upper bounded by $O(\sqrt{n})$, which implies a $O((\log n)^2/\sqrt{n})$ per-node capacity in case of random networks. Thus, to the best of our knowledge, unicast communications in wireless multi-hop networks *are* relay limited, and per-node capacity must be traded off with packet delivery latency.

Investigation of fundamental properties of broadcast communications has received attention from the research community only very recently. In [16], Zheng investigated the broadcast capacity for the case of random networks with single broadcast source under the generalized physical interference model, and presented a broadcast scheme providing a capacity within a constant factor from optimal. The authors of [9] considered a more general network model, in which arbitrary node positions are allowed, and an arbitrary subset of the network nodes is assumed to generate broadcast packets. The results of [9] confirms the findings of [16], i.e., that the (aggregate) broadcast capacity is within a constant factor from the optimal capacity. More recently, the same authors have proved in [8] that the same result holds using more realistic interference models, namely the physical and the generalized physical interference model. However, none of the existing work on broadcast capacity has investigated the *latency* in message delivery. More specifically, in the network models used in [8], [9], [16] it is assumed that broadcast packets are *eventually* received by all network nodes, but no explicit nor implicit upper bound on delivery time is given. Zheng in [16] also investigated the information diffusion rate, which is closely related to latency, and provides matching upper and lower bounds for this quantity. However, the broadcast schemes used for lower bounding broadcast capacity and information diffusion rate are different, and the problem of finding an optimal broadcasting scheme for both capacity and information diffusion rate in the model of [16] remains open.

The above discussion highlights that the fundamental question of whether asymptotically optimal capacity and latency can be achieved simultaneously in case of broadcast communication remains, to the best of our knowledge, open.

In this paper, we give a positive answer to this question by showing that, at least under certain, quite general, assumptions on node deployment, *asymptotically optimal capacity and latency can indeed be achieved simultaneously*. To derive this result, we introduce a novel topological notion of network connectivity which we call *backward connectivity*, and show that asymptotically optimal broadcast capacity and latency can be achieved simultaneously if the network is backward connected. Our result is proved using the realistic physical interference model of [7], and under the assumption of constant number of broadcast sources. Thus, in sharp contrast with the case of unicast transmission, our result suggests that *broadcasting is not relay limited*, and scalable broadcasting in wireless multi-hop networks is, in principle, possible.

We stress that latency is a fundamental parameter of broadcast communication, at least in some scenarios such as multimedia and real-time applications. For instance, if a wireless multi-hop network is used for communication of multimedia information among members of a disaster rescue team.

The problem of latency optimal broadcasting (with no consideration on broadcast capacity, though) is studied, for instance, in [5].

II. Network model and preliminaries

We consider a wireless network composed of n wireless nodes distributed in a two-dimensional domain. We assume nodes communicate through a shared wireless channel of a certain, constant capacity W, and that the nodes transmission power is fixed to some value P. Correct message reception at a receiver node is subject to a SINR-based criterion, also known as *physical interference model* [7]. More specifically, a packet sent by node u is correctly received at a node v (with rate W) if and only if

$$\frac{P_v(u)}{N + \sum_{i \in \mathcal{T}} P_v(i)} \geq \beta \ ,$$

where N is the background noise, β is the capture threshold, $\mathcal{T}$ is the set of nodes transmitting concurrently with node u, and $P_v(x)$ is the received power at node v of the signal transmitted by node x.

We also make the standard assumption that radio signal propagation obeys the log-distance path loss model [12], according to which the received signal strength at distance d from the transmitter (for sufficiently large d, say, $d \geq 1$) equals $P \cdot d^{-\alpha}$, where α is the path loss exponent. In the following, we make the standard assumption that $\alpha > 2$, which is often the case in practice. We then have[1] $P_v(x) = P \cdot d(x,v)^{-\alpha}$, where $d(x,v)$ is the Euclidean distance between nodes v and x, and the SINR value at node v can be rewritten as follows

$$SINR(v) = \frac{d(u,v)^{-\alpha}}{\frac{N}{P} + \sum_{i \in \mathcal{T}} d(i,v)^{-\alpha}} \ .$$

For given values of P, β, α, and N, we define the transmission range r_{max} of a node as the maximum distance up to which a receiver can successfully receive a message *in absence of interference*. From the definition of physical interference model, we have $r_{max} = \sqrt[\alpha]{P/(\beta N)}$.

The *maximal communication graph* is a graph $G_M = (\mathcal{V}, \mathcal{E}_{\mathcal{M}})$ representing all possible communication links in the network, i.e., (undirected) edge $(u,v) \in \mathcal{E}$ if and only if $d(u,v) \leq r_{max}$.

Given an arbitrary broadcast source node s in a network with n nodes, we define the *broadcast capacity* of the network as the maximum possible rate $\lambda(n)$ at which the source can generate packets such that all generated packets are received by all nodes in $\mathcal{V} - \{s\}$ within a certain time T_{max}, with $T_{max} < \infty$. In case of multiple broadcast source nodes, $\lambda(n)$ refers to the maximum *aggregate* rate at which source nodes can generate packets such that all generated packets are correctly received by the other nodes. The *broadcast latency* of the network is the minimal time $T(n)$ such that all nodes in $\mathcal{V} - \{s\}$ receives a packet generated by a source node s at time t within time $t + T(n)$. It is clear that, in order to have meaningful values of broadcast capacity and latency, the maximal communication graph of a network must be connected. Thus, the assumption of connected maximal communication graph is made throughout this paper.

III. Backward connected networks

In order to obtain non-trivial bounds for the broadcast latency, some assumptions on network deployment must be made. In fact, while broadcast capacity is somewhat independent of the shape of the deployment region and node positions as long as the maximal communication graph is connected (see, e.g., [8]), broadcast latency depends on the distance between the broadcast source and the farthest node, which, in turn, depends on how nodes are deployed in the plane.

In the following we introduce the notion of *backward connectivity*, which is sufficient to obtain asymptotically optimal upper bounds to broadcast latency. Before introducing backward connectivity, we define a square lattice of the plane, and introduce the notion of *cell distance* between an arbitrary pair of network nodes.

Definition 1 (Cell graph): Assume the plane is partitioned into a lattice of square cells of side l, with $l = \frac{r_{max}}{2h\sqrt{2}}$, for some constant $h > 1$, and let $v_1, \ldots, v_n$ denote the positions of the n network nodes in the plane. The cell graph of the network is the graph $CG = (\mathcal{C}, \mathcal{EC})$, with a vertex corresponding to each cell c such that $c \cap CH(\mathcal{V}) \neq \emptyset$, with $CH(\mathcal{V})$ denoting the convex hull of points $v_1, \ldots, v_n$, and undirected edge $(x,y) \in \mathcal{EC}$ if and only if cells x, y are adjacent (horizontal, vertical, and diagonal adjacency).

Definition 2 (Cell distance): Let $cell(x)$ denotes the cell to which node $x \in \mathcal{V}$ belongs to. The cell distance $cd(u,v)$

[1] To simplify notation, in the following we assume that the product of the transmitter and receiver antenna gain is 1.

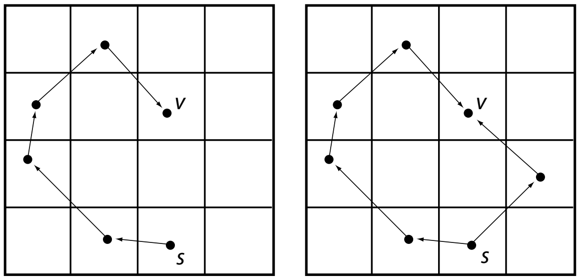

Fig. 1. The notion of backward connectivity.

between nodes $u, v \in \mathcal{V}$ is defined as the hop distance between the corresponding vertexes $cell(u)$ and $cell(v)$ in the cell graph.

Claim 1: Let u be an arbitrary node located in cell $cell(u)$; all the nodes located in the 8 cells adjacent to $cell(u)$ are within distance r_{max} from u.

In the following, we use the term *communication graph* (or, simply, graph) to refer to an arbitrary subgraph $G = (\mathcal{V}, \mathcal{E})$ of the maximal communication graph G_M. Intuitively speaking, a communication graph represents the set of links used by a certain communication scheme, which does not necessarily include all possible links in the network. Unless otherwise stated, in the following $(u, v) \in \mathcal{E}$ denotes a *directed* transmission link between node u (the sender, or transmitter, node) and node v (the receiver node).

Definition 3 (Backward connectivity): Let $G = (\mathcal{V}, \mathcal{E})$ be an arbitrary communication graph. Graph G is backward connected with respect to node $s \in \mathcal{V}$ if and only if, for any node $v \in \mathcal{V} - \{s\}$, there exists at least one path P connecting s and v in G such that for all nodes w in $P - \{v\}$, we have $cd(s, w) < cd(s, v)$.

The notion of backward connectivity is pictorially explained in Figure 1: the network on the left is not backward connected w.r.t. s[2], since the only path connecting s and v contains nodes whose cell distance to s is greater than $cd(s, v) - 1 = 1$. On the contrary, the network on the right is backward connected, since the rightward path connects s to v through a node whose cell distance to s is less than $cd(s, v)$. As we shall see, the notion of backward connectivity is fundamental to ensure an asymptotically optimal progress of broadcast packets generated at s towards nodes in $\mathcal{V} - \{s\}$, i.e., optimal broadcast latency. Note also that the notion of backward connectivity is a monotonic graph property.

A communication graph $G = (\mathcal{V}, \mathcal{E})$ is said to satisfy the *cell adjacency* property if and only if set $\mathcal{E}$ is a superset of the set of links (u, v) with nodes u, v belonging to either adjacent (horizontal, vertical, and diagonal adjacency) cells or to the same cell. In other words, the cell adjacency property implies that all links whose endpoints lie in the same or adjacent cells are part of the communication graph.

Given the above property, the following claims are trivial to show:

[2]When the actual node s w.r.t. a graph G is backward connected is not relevant, we will simply say that G is “backward connected”.

Claim 2: If every cell in the cell graph corresponding to a certain communication graph $G = (\mathcal{V}, \mathcal{E})$ satisfying cell adjacency contains at least one node, then graph G is backward connected w.r.t. any node $v \in \mathcal{V}$.

Claim 3: If every empty cell in the cell graph corresponding to a certain communication graph $G = (\mathcal{V}, \mathcal{E})$ satisfying cell adjacency is adjacent only to non-empty cells, then graph G is backward connected w.r.t. any node $v \in \mathcal{V}$.

Note that the number and patterns of empty cells depends on the the step l of the square lattice used to partition the plane. Intuitively speaking, fixed arbitrary positions for the network nodes, the larger the step l of the square lattice, the less likely it is to have empty cells. On the other hand, the larger the value of l, the smaller the cell distance between arbitrary nodes tends to be, hindering existence of “backward connected” paths in the communication graph. We have verified through simulations with random node deployment (not reported for lack of space) that, even for low density scenarios, the vast majority (above 98%) of networks such that the maxpower communication graph is connected are also backward connected.

To ease presentation, in the following we consider a network to be backward connected w.r.t. a certain node s if there exists *at least one* square lattice partitioning of side $\bar{l} = \frac{r_{max}}{2\bar{h}\sqrt{2}}$, where $\bar{h}$ is a constant greater than 1, such that the network is backward connected w.r.t. node s for that specific lattice. This specific square lattice (or one of them, in case the network is backward connected for more than one square lattices) is assumed to be used in the derivation of optimal broadcast capacity and latency bounds.

We conclude this section with the proof that a relevant network deployment scenario, known as *homogeneous networks* in the literature, satisfies backward connectivity w.r.t. any node u. More specifically, we consider the following node deployment scenario:

$a1$. a number n of nodes is distributed uniformly at random in a square region of side $(r_{max}/j) \cdot \sqrt{n/\log n}$, where j is a properly defined constant.

In the above setting, the critical transmission range for connectivity $ctr(n)$, i.e., the minimal value of the transmission range such that the resulting network topology is connected w.h.p.[3] (this is a necessary condition for having meaningful notions of broadcast capacity and latency) is [3], [8]:

$$ctr(n) = \sqrt{\left(\frac{r_{max}}{j}\right)^2 \cdot \frac{n}{\log n} \cdot \frac{\log n}{n}} = \frac{r_{max}}{j} ,$$

i.e., it is a constant fraction of the maximum transmission range. In other words, we are considering a situation in which node density (defined here as the average number of nodes within transmission range) is minimal (up to a constant factor j) for obtaining network connectivity w.h.p..

Let us now assume the deployment region is divided into $C = \frac{8h}{j} \cdot \frac{n}{\log n}$ non-overlapping square cells of side $l = \frac{r_{max}}{2h\sqrt{2}}$, where $h > 1$ is an arbitrary constant.

[3]In this paper, w.h.p. means with probability approaching 1 as $n \to \infty$.

We now prove a concentration result that shows that, under the above conditions, *all* the cells contains at least one node w.h.p., which, by Claim 2, implies backward connectivity w.r.t. any node s under the assumption that the communication graph satisfies cell adjacency.

Lemma 1: Assume n nodes are distributed uniformly at random in a square region of side $\frac{r_{max}}{j} \cdot \sqrt{\frac{n}{\log n}}$, for some arbitrary constant j, and that the deployment region is divided into $C = \frac{8h}{j} \cdot \frac{n}{\log n}$ non-overlapping square cells of side $l = \frac{r_{max}}{2h\sqrt{2}}$. If $j \geq 8h$, then the minimally occupied cell contains at least one node, w.h.p.

Proof: The proof is omitted for lack of space. See [13]. ■

Corollary 1: Assume $a1$; if a communication graph G satisfies cell adjacency, then G is backward connected w.r.t. any node s w.h.p.

IV. Trivial bounds on broadcast capacity and latency

The following upper bound on the broadcast capacity trivially follows by observing that the maximum rate at which any receiver can receive broadcast packets is W [8]. The bound holds for an arbitrary network.

Claim 4: In any network with n nodes, we have $\lambda(n) \leq W$.

Define $D(n)$, the *diameter* of the network, as the maximum Euclidean distance between any two network nodes $u, v \in \mathcal{V}$. The lower bound on the broadcast latency immediately follows by the definition of transmission range r_{max}. Also this bound holds for an arbitrary network.

Claim 5: Given any network with n nodes, we have $T(n) \geq \frac{D(n)}{r_{max}}$.

V. Matching capacity and latency bounds

In this section, we introduce a broadcast scheme for single source broadcasting based on a k^2 coloring of a subset of network nodes, which enjoys the following properties: $i)$ the broadcast source s generates new packets at rate $\Omega\left(\frac{W}{k^2}\right)$; and $ii)$ all generated packets are correctly received by all nodes in $\mathcal{V} - \{s\}$ within time $O\left(\frac{D(n)}{r_{max}}\right)$ under the condition that a properly defined communication graph G_k is backward connected w.r.t. source node s.

Note that, in order to have asymptotically optimal broadcast capacity, the number of colors used by the broadcast scheme must be a constant. On the other hand, we shall see that, fixed a step l of the square lattice used to partition the plane, $k' < k$ implies that communication graph $G_{k'}$ is a subgraph of G_k. Given that backward connectivity is a monotonic graph property, $G_{k'} \subseteq G_k$ implies that, from the point of view of backward connectivity (which is a sufficient condition to show asymptotically optimal broadcast latency), a relatively large value of k is desirable. In other words, if we select a relatively low value of k, it is relatively more likely that the resulting communication graph G_k is not backward connected, and optimal latency in packet delivery cannot be ensured. On the other hand, a very large value of k (say, $k = f(n)$, where $f(n)$ is some unbounded increasing function of n) is likely to result in a backward connected communication graph, but it is not optimal for what concern broadcast capacity.

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Algorithm for a generic node v:
    Let i be the color of the current time slot
    If v is a leader node, let j be the ID of the
    last packet transmitted by node v
 if color(v) = i then
     if source(v) then transmit new packet
     else if cellLeader(v) then
         if buffer(v) is not empty then
             transmit packet and empty buffer
 else  // color(v) ≠ i
 if not source(v) then
     listen to the channel
     if new packet arrive then
         receive the packet
         let j' be the ID of the received packet
         if (cellLeader(v)) and (j' = j + 1) then
             store packet in transmit buffer
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Fig. 2. The broadcasting scheme.

In this section, we show how to address this tradeoff, by showing that there exists a *constant* value $\bar{k}$ such that communication graph $G_{\bar{k}}$ satisfies cell adjacency, which ensures that backward connectivity of graph $G_{\bar{k}}$ can be *guaranteed* for the relevant network deployments characterized in claims 2 and 3 and Corollary 1. More in general, our characterization shows that asymptotically optimal broadcast capacity and latency is achievable whenever communication graph $G_{\bar{k}}$ is backward connected w.r.t. the source node, which can occur also when none of the conditions necessary for claims 2 and 3 and Corollary 1 holds.

To prove our result we use the simple scheme reported in Figure 2. We assume that the plane is partitioned into non-overlapping square cells of side l, that each node v is aware of the cell $cell(v)$ to which it belongs, and that a TDMA scheme is used at the MAC layer. Each node v in the network is assigned with a color $color(v)$ chosen among a set of k^2 colors. Details of the coloring scheme, which is similar to the ones used, e.g., in [8], [16], are given below. Time is divided into periods composed of k^2 transmission slots, one for each color. We assume slot coloring is periodic with period k^2, i.e., if slot j has color i, then also slots $j + zk^2$, for any integer $z \geq 1$, have color i. All nodes have a single-packet transmit buffer; i.e., when a new packet arrives, if the previous packet has not yet been sent, the old packet is overwritten.

The source node s simply transmits a new packet each time a slot of color $color(s)$ occurs. Leader nodes are selected arbitrarily in each populated cell. If a node v is selected as cell leader, i.e., as the only node in $cell(v)$ responsible for forwarding packets, then v transmits a packet whenever a slot of color $color(v)$ occurs, subject to the condition that there is a new packet to send in the buffer. Each non-source node listens to the channel in the remaining $k^2 - 1$ slots of the period and, in case a new packet is received and the node is a cell leader, the new packet is stored in the buffer (possibly overwriting an old packet). Note that new packets can be easily identified by a sequential packet ID contained in the header of the packet.

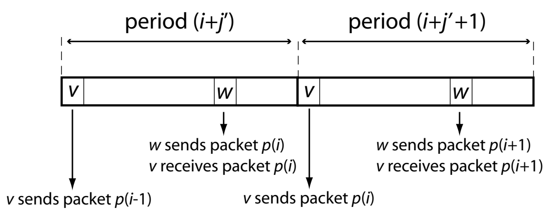

Fig. 3. Transmission opportunities in consecutive periods.

Condition $j' = j + 1$ at step 12. ensures that the broadcasting scheme preserves packet ordering, i.e., single packet buffering at the leader nodes is sufficient (see Theorem 1 for details).

The coloring scheme is as follows. We recall that we assume a square lattice partitioning as described in Section III. The cells have side l, and we assume that their bottom left corners have coordinates $(i \cdot l, j \cdot l)$ for $i, j \in \mathbb{Z}$. The cells are colored in a checkered fashion with k^2 colors. Each color can be identified by a pair (a, b), with $a, b \in \{0, 1, \ldots k-1\}$; then the cell whose bottom left corner has coordinates $(i \cdot l, j \cdot l)$ has color $(i, j) \pmod k$. For each node v, $color(v)$ is the color of the cell it belongs to. In the following we assume, w.l.o.g., that $k \geq 3$.

Given a value of k, we define communication graph G_k as follows:

Definition 4 (graph G_k): Communication graph G_k includes all and only directed links (u, v) such that u is either the source or a leader node, and the SINR value at node v relative to the signal transmitted by node u is at least β, under the assumption that *all* leader (or source) nodes of color $color(u)$ are transmitting simultaneously.

In other words, graph G_k contains all and only the links that can be used to broadcast packets when the broadcasting scheme in Figure 2 is used and the number of colors is k^2.

We now prove some fundamental properties of the broadcasting scheme in Figure 2:

Theorem 1: The broadcasting scheme defined in Figure 2 satisfies the following properties:
$i)$ the source s generates broadcast packets with rate $\frac{W}{k^2}$;
$ii)$ let $p(i)$ be the packet generated by node s during period i; if communication graph G_k is backward connected w.r.t. s, packet $p(i)$ is received by all nodes in $\mathcal{V} - \{s\}$ within time $O(\frac{D(n)}{r_{max}})$.

Proof: We recall that we are assuming a time-slotted approach at the MAC layer, that time slots are colored using $\bar{k}^2$ colors, and that slot coloring is periodic with period $\bar{k}^2$. Property $i)$ is straightforward: the source node s generates a new broadcast packet each time a slot with color $color(s)$ occurs, and thus gets $\frac{1}{k^2}$ of the available channel capacity W. To prove property $ii)$, we first show that $a)$ any leader node at cell distance $j \geq 0$ from the source node s transmits packet $p(i)$ within period $i + j$. We proceed by induction on j. The base case $j = 0$ (i.e., the transmitting node is the source) is straightforward. Let us now consider the inductive case. Let v be a leader node at cell distance $j + 1$ from the source. By backward connectivity, there exists a path P in the communication graph G_k such that all nodes in $P - \{v\}$ have cell distance at most j. Observe that, by definition of G_k, path P is composed only of leader nodes. Assume w.l.o.g. that path P has minimal hop-length among the paths satisfying the condition for backward connectivity. Let w denote the only node in P such that link (w, v) is in G_k. Since $cd(w, s) \leq j$, by induction hypothesis node w transmits packet $p(i)$ within period $i + j'$, with $j' \leq j$. Since edge (w, v) is in G_k, packet $p(i)$ is correctly received by node v during period $i + j'$. Hence, when node v has its own transmission opportunity during period $i + j' + 1 \leq i + j + 1$, packet $p(i)$ is in the transmission buffer and is transmitted (see Figure 3). Note that step 12. of the broadcasting scheme requires that node v has transmitted packet $p(i-1)$ before being able to store packet $p(i)$ in the transmit buffer. This is actually the case, since, by induction hypothesis, packet $p(i-1)$ is transmitted by node v during period $i + j'$. We then conclude that node v transmits packet $p(i)$ within period $i + j + 1$, and property $a)$ is proved. The following property immediately follows from property $a)$ above and backward connectivity: $b)$ packet $p(i)$ generated by source node s during period i is received by all nodes at cell distance $\leq j$ from s by the end of period $i + j - 1$, for any $j \geq 1$. Property $ii)$ now follows from $b)$, by observing that the maximum (Euclidean) distance between s and any other node in $\mathcal{V} - \{s\}$ is at most $D(n)$, and that, given that the square lattice step l is within a constant factor from r_{max}, the cell distance between two nodes at Euclidean distance d is $O(\frac{d}{r_{max}}) = O(\frac{D(n)}{r_{max}})$. ■

We now prove the following fundamental lemma, which shows that there exists a *constant* value $\bar{k}$ of the number of colors such that any packet sent by a node in our broadcasting scheme is correctly received by *all* the nodes in the 8 surrounding cells. This implies that communication graph $G_{\bar{k}}$ satisfies cell adjacency, and backward connectivity of graph $G_{\bar{k}}$ is *guaranteed* in network deployments characterized in claims 2 and 3 and Corollary 1.

Lemma 2: Assume a cell partitioning as defined in Section III. If $k \geq \bar{k} = \left\lceil 2 + 2^{\frac{3}{2} + \frac{4}{\alpha}} \left(\beta\zeta(\alpha - 1)h^{\alpha}/(h^{\alpha} - 1)\right)^{\frac{1}{\alpha}} \right\rceil$, where ζ is the Riemann's zeta function, then every packet sent by a node in the broadcasting scheme is correctly received by all nodes in adjacent cells.

Proof: Proof omitted for lack of space. See [13]. ■

Note that conditions $h > 1$ and $\alpha > 2$ ensure that the value of $\bar{k}$ is finite and, in accordance with the theoretical findings of [8], the value of $\bar{k}$ is independent of node density, but depends only on the step of the square lattice and on the path-loss exponent.

By observing that the capacity and latency bounds provided by our broadcasting scheme match the corresponding bounds stated in Section IV when $k = \bar{k}$, we have the following theorem, which is the main result of this paper:

Theorem 2: The broadcasting scheme defined in Figure 2 provides asymptotically optimal broadcast capacity and latency under the assumption that graph $G_{\bar{k}}$ is backward connected w.r.t. the source node s.

The scheme can be trivially extended to provide optimal broadcast capacity and latency bounds in case of multiple source nodes $s_1, \dots, s_z$, where z is an arbitrary *constant*. The idea is to separate the z concurrent broadcasts into non-overlapping periods, i.e., to broadcast packets generated by source s_i during period $i, i+z, i+2z, \dots,$ for $1 \le i \le z$. It is immediate to see that the achieved aggregate broadcast rate is unchanged, while packet latency is reduced by a constant factor z, i.e., it remains asymptotically optimal. We can then conclude this section with the following theorem:

Theorem 3: The broadcasting scheme defined in Figure 2 provides asymptotically optimal broadcast capacity and latency in a network of n nodes with z sources $s_1, \dots, s_z$, where $z \ge 1$ is an arbitrary constant, under the assumption that graph $G_{\bar{k}}$ is backward connected w.r.t. nodes $s_1, \dots, s_z$.

Observe that the above construction naturally brings a tradeoff between broadcast capacity and latency when the number of source nodes is a function of n. For instance, with $\log n$ sources, we have asymptotically optimal capacity, but latency which is within a $O(\log n)$ factor from optimal.

VI. Discussion

The results presented in this paper have shown that, contrary to what happens in case of unicast transmissions, broadcasting in wireless multi-hop networks *is not* relay limited. This discrepancy originates from the fact that a single wireless communication is potentially correctly received by all nodes within transmission range. While in case of unicast *only one* of these potentially many receivers is actually interested in the packet, and all the other nodes treat the incoming signal as interference, in case of broadcast *all* potential receivers are interested in receiving the packets.

The fact that unicast transmissions are relay, and not interference, limited, is supported by well-known results (see, e.g., Lemma 2, and similar results in [8], [9], [16]) that $\Theta(n)$ simultaneous successful transmissions can occur in a wireless network, i.e., interference limits the degree of spatial reuse at most by a constant factor. However, in case of unicast communication these transmissions can be used to move relatively few packets toward destination, due to the additional relay burden on the nodes. It is not then surprising that, if the relay burden is limited (e.g., only 1-hop flows occurring in the network) asymptotically optimal per-node capacity (and, trivially, latency) can be achieved also with unicast transmissions [7]. Observe that lessening the relay burden through exploiting node mobility is at the basis of Grossglauser and Tse's result [6] which, however, comes at the expense of increasing packet latency. More recently, Ozgur et al. [11] have suggested using cooperative transmissions to lessen the relay burden on the nodes. However, this approach can be used to improve unicast capacity up to a suboptimal factor, due to physical limitations to the maximum number of independent information channels crossing a two-dimensional domain [4]. Thus, the relay burden in case of unicast communications cannot be reduced up to a level where asymptotical optimal capacity and latency can be achieved simultaneously. As shown in this paper, this is not the case of broadcast communication.

VII. Conclusions

In this paper, we have investigated the fundamental question of whether asymptotically optimal broadcast capacity and latency can be simultaneously achieved in a wireless multi-hop network. To answer this question, we have introduced a novel topological notion of graph connectivity, called backward connectivity, and shown that asymptotically optimal capacity and latency can be achieved in a backward connected network.

Several issues are left open by this paper, such as investigating whether our result can be extended to the case of a non-constant number of broadcast sources or to the generalized physical interference model. Furthermore, a question of interest is understanding whether backward connectivity, which is a sufficient condition for having asymptotically optimal broadcast latency, is also a necessary condition.

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