Computational Geometry

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Lecture 5: point-in-polygon

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Summary of lecture 5

- Summary of lecture 4:
  Making shapes,
  Convex Hull
  Jarvis's March, Graham's scan.
- Summary of lecture 5.
  Searching in simple polygons:
  point-in-polygon queries.
  Searching in convex polygons:
  point-in-polygon queries.

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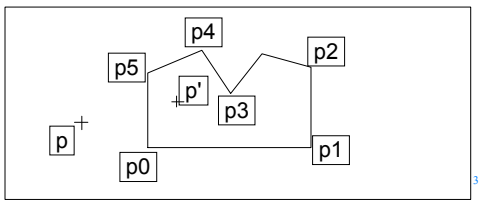
Point-in-polygon queries

(for simple polygons)

- We are given a simple polygon P as a list of its vertices (p0,p1,...,pn).
- Afterwards a new point p is given (e.g. by clicking the mouse on the screen.



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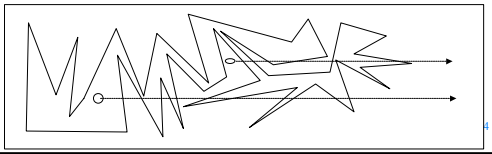
Point-in-polygon, 2.

- If we click inside the polygon P we expect the computer to perform some action; if we click outside we expect a different action.
- So we have to decide the point-in-polygon query.
- The *crossing-number* method:



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The ray crossings method

- The idea of this method is to COUNT how many edges of P are crossed by a ray starting from the query point p and directed along the positive x direction.
- If we do not hit any vertex of P, then when p is outside we get an even number of crossings, when p is inside we get an odd number of crossings.
- Special cases: the ray pass through a vertex, the ray overlaps an edge.

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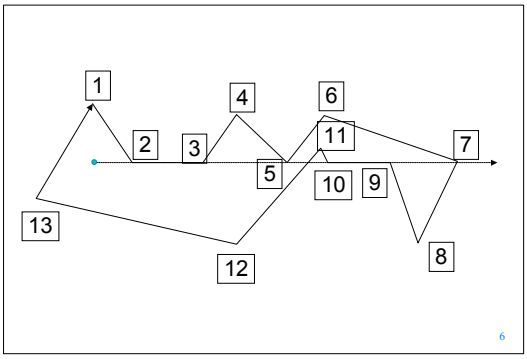
Special cases



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How to deal with the special cases

- If the ray R overlays the edge (p[i],p[i+1]) we check the points p[i-1] and p[i+2], if they are on the same side of R we do not have a crossing, if they are on different sides we have a crossing.
- If the ray R intersect edge (p[i],p[i+1]) at p[i+1], then we have to check whether p[i+2] and p[i] are on different sides of the ray, in such case we have a crossing.
- We need to modify the segment intersection detection procedure so that it returns more than TRUE/FALSE: we need the values: NO-INTERSECTION, OVERLAY, HIT-SECOND-VERTEX, PROPER-INTERSECTION.
- We can give R by a pair (p, (+infinity,py)).

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Point-in-simple-polygon-test

- Point-in-polygon((p,q): ray; P[0,..,n-1]: Polygon)
  Cross:= 0;
  FOR i:=0 TO n-1
  DO x := INTERSECT((p,q),(P[i],P[i+1])
  CASE x OF
  PROPER-INT : Cross:=Cross+1
  NO-INT : do nothing
  HIT-SECOND-V :
  IF (left(p,q,P[i]) ≠ left(p,q,P[i+2]))
  THEN Cross:=Cross+1
  END IF
  OVERLAY:
  IF (left(p,q,P[i-1]) ≠ left(p,q,P[i+2]))
  THEN Cross := Cross+1
  END IF
  END FOR
  IF EVEN(Cross) THEN RETURN False,
  ELSE RETURN True
  END IF

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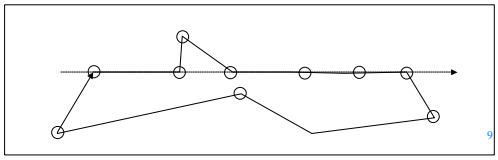
Observations and exercise

- We assume that all index arithmetic is done MOD n, so to be able to wrap around the array.
- The code does work properly only if we assume that there are not 2 collinear consecutive edges.
- Exercise: modify so to make it work when there are collinear consecutive edges.



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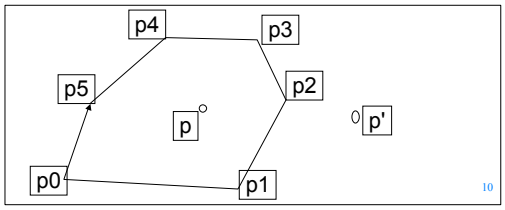
When the input polygon is convex.

- The point-in-polygon procedure for simple polygons works in time O(n). This procedure works for convex polygons too.
- For convex polygons we can produce simpler code by exploiting some properties of convex polygons. Any suggestion?



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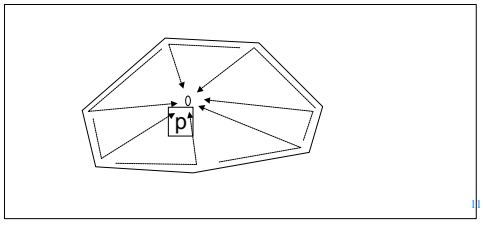
Point-in-convex-polygon

- We can exploit the fact that if p is inside a convex polygon C=[p0,...,pn] then it makes always the same type of turn with the edges of the polygon.



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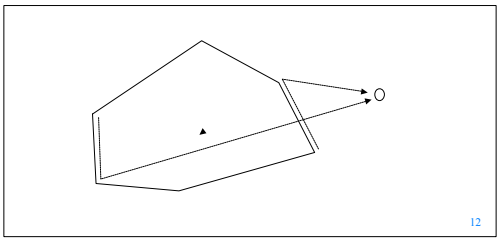
Point-in-convex-polygon, 2

- Instead if p is outside the convex polygon C, then we will always find two different turns while we visit the edges of C one by one in (clock-wise or counterclockwise) sequence.



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Exercise

- Can you write the code to test is a point p is inside or outside the a convex polygon C =[p0,...,pn]?
- You should USE the procedures left(..), collinear(..) etc, that we introduced in the previous lecture.
- Can you handle the case when p in on the boundary of C?
- What is the time complexity of your procedure?

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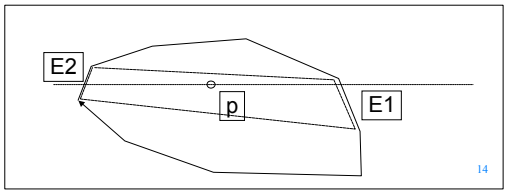
A fast point-in-convex-polygon test

(using binary search)

- Main idea: if p is in C then when we draw an horizontal line through p we hit two edges E1, E2. p is also inside the convex quadrilateral formed by the vertices of E1 and E2.



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Fast point-in-convex-polygon test, 2

- Conversely, if p is outside Q(E1,E2) then it is outside C.
- The strategy is: find E1 and E2, then apply the previous method to the quadrilateral Q(E1,E2).
- To start with we find the points with the largest and smallest y-coordinate. We find the right chain and the left-chain. E1 is in the left chain, and E2 is in the right chain, so we need to describe only how we find one of them, the other is found using the same procedure.

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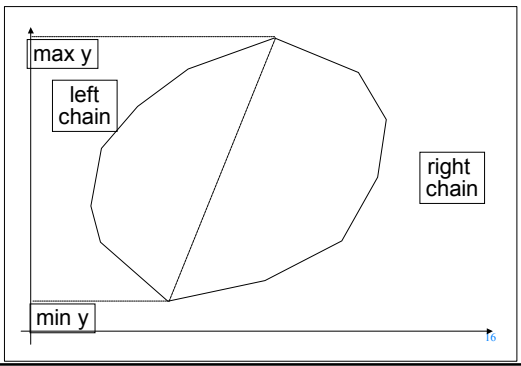
Left chain, right chain



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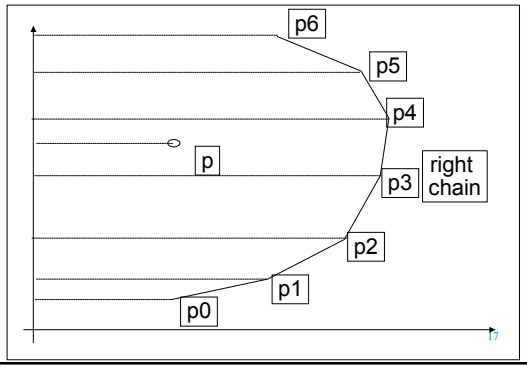
Right chain only



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Determining E1.

- We make a binary search with py on the array of numbers [p0y,p1y,p2y,p3y,p4y,p4y,p5y,p6y], which are the y coordinates of the vertices of the right chain. This array is sorted.
- We locate px in the array by binary search and we find the two neighbours: p3y and p4y. Then the horizontal line through p hits the edge (p3,p4) and this is the edge E1 we were looking for.
- Since we use binary search, to find E1 takes time O(log n), after the left chain and the right chain have been found.

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Fast point-in-convex-polygon

(pseudocode)

- Preprocessing(C: convex polygon)
  Find top vertex,
  Find bottom vertex,
  Find left chain.
  Find right chain.
- Fast-in-convex-polygon(p: point, C: convex polygon)
  Use binary-search on left-chain with py to determine E1.
  Use binary search on right-chain with py to determine E2
  Find quadrilateral Q(E1,E2).
  RETURN(Slow-point-in-convex-polygon(p,Q)).

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Observation and exercises

- Exercise, what is the running time of the prerpocessing on a convex polygon of n points?
- What is the running time of 1 in-polygon query?
- How much storage do we need? Can we do everything in place (i.e. using the original array of points of C)?

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Conclusions

- We have seen how to test whether a query point is inside or outside a simple polygon
- We have seen a second easy method that works in linear time for convex polygons.
- Finally we have seen a fast method for testing whether a point is inside or outside a convex polygon that uses time O(log n) after linear preprocessing.
- Now we can click a mouse on an a drawing represented as a (simple, convex) polygon.

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Homework

(due in 3 weeks)

- Write a program in C or Pascale to test whether a point is inside or outside a simple polygon.
- Test the code with a polygon with 20 vertices in such way to test all the special conditions discussed in class.
- Test at least 20 points for inside/outside.
- Make a drawing of the polygon and the points and compare the drawing with the output of your program.

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Further readings

- http://www.ecse.rpi.edu/Homepages/wrf/Research/Short_Notes/pnpoly.html
- http://astronomy.swin.edu.au/~pbourke/geometry/insidepoly/
- http://softsurfer.com/Archive/algorithm_0103/algorithm_0103.htm

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